ON THE OPTIMAL RECEIVER ACTIVATION FUNCTION FOR DISTANCE-BASED GEOGRAPHIC TRANSMISSIONS

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Abstract. In wireless networks, the channels are often subject to random variations that limit the reliability of communications between any two radios. Geographic transmission strategies can improve the performance in such networks, by allowing any of a transmitter's neighbors that successfully receive a transmission and are in the direction of the packet's destination to forward the packet on to the destination. However, requiring all of the radios in a network to keep their receivers on to receive geographic transmissions will significantly shorten the network lifetime by depleting the energy of the radios in the network. In this work, we investigate an optimal strategy for deciding which neighbors of a transmitter should activate to try to recover a transmission. We find a solution to this problem by solving for a related measure in a constrained optimization problem. We present results that compare the performance of the optimal approach, our previous suboptimal efforts, and a conventional approach.

Key words. measure optimization, functional optimization, geographic transmission, fading channels, multiuser diversity

AMS subject classifications. 90B18, 90B15, 46N10, 94A05, 94A40

1. Introduction.

In wireless mobile ad hoc networks (MANETs), there are no fixed access points or infrastructure, and radios communicate on a peer-to-peer basis [17]. These networks are well suited for situations in which network infrastructure is undesirable, for instance when there is insufficient time to deploy such resources. Such scenarios typically arise in tactical military communications and during disaster-recovery operations. The communication signals in wireless MANETs usually experience significant losses from both electromagnetic attenuation over distance and *multipath fading*. Multipath fading, often just called fading, is a random phenomenon caused when the transmitted signal reflects off objects in the environment. The signal at the receiver is a superposition of the reflected versions of the signal, which undergo random delays and attenuations. Fading has traditionally been treated as an unpredictable phenomenon that degrades the performance of communication systems. However, recent advances in wireless communication research have shown that random fading in the presence of several users can actually be utilized opportunistically to improve the system performance [12, 19]. This improvement is known as the *multiuser diversity benefit* in wireless communications. Third-generation (3G) cellular communication systems have already been deployed that make use of multiuser diversity benefit [1, 6].

The lack of centralized control makes it more difficult to achieve multiuser diversity in MANETs. Nevertheless several schemes [13, 14, 20, 25, 24, 21] have been proposed that achieve multiuser diversity benefit in MANETs, following approaches similar to [12]. A common assumption in all of these schemes is that the transmitter has complete or partial knowledge of the channel state information at the receiver. However, such information may not be available, or may not be accurate, in MANETs

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because the channel may change significantly between the times the channel is estimated.

In a MANET, messages that are not intended for a neighbor of a source radio are routed to the destination by the other radios in the network. In the conventional approaches to routing, at each hop along the route, the transmitting radio will preselect one of its neighbors to act as the next-hop forwarding agent for the packet. However, in the presence of fading, it is likely the channel state at any pre-selected radio will not be sufficiently good to recover the packet, which implies the packet will have to be retransmitted or re-routed. Thus, approaches that pre-select a single receiver or even multiple receivers as the next-hop forwarding agent for a message may offer poor performance if the particular receiver(s) experience a bad channel. To avoid this difficulty the use of *geographic transmission* has been suggested [28, 27, 29, 22, 26, 25, 10, 7]. The fundamental philosophy behind geographic transmissions is that any of a group of receivers that are located in the "direction" of the destination can choose to be the next-hop forwarding agent. Moreover, geographic approaches have been shown to provide multiuser diversity benefit in fading channels (cf. [8] and the references therein).

However, geographic transmission requires that a radio's neighbors must keep their receivers active during that radio's transmissions. This can have a detrimental effect on network lifetime, as mobile radios have limited battery energy, and the energy consumed in receiving messages can be comparable to that used in transmitting messages [4, 5]. As a result, several authors have suggested receiver-activation techniques (powering off some of the redundant receivers) to conserve energy [3, 2, 28, 29, 27, 7]. In all of these works, heuristic techniques to conserve energy have been discussed that are not based on any optimization criterion.

In this paper, we consider the problem of determining which of a radio's neighbors should keep their receivers active to try to recover a message under a constraint on the total energy consumed in receiving a transmission. In particular, we formulate this as a constrained optimization problem in which we wish to maximize the maximum of the distances to the successful receivers under a constraint on the expected number of radios that activate to receive the message. The neighboring radios are assumed to be distributed according to a two-dimensional Poisson point process, and their locations are unknown to the transmitter. However, we assume that the radios can use knowledge of their own location and the location of the transmitter in deciding whether to activate to receive a packet. Since we only consider transmission distance in this work, then the radios activate probabilistically based on their distance from the transmitter. We call this approach node-activation based on link distance (NA-BOLD). The goal is to find the optimal node-activation function, which is the conditional probability that a radio should activate given the distance of that radio from the transmitter. We originally described this general approach and provided a suboptimal solution in [9]. Additional details of the suboptimal solution and a computationally feasible method to approximate the solution are given in [8]. We show in [10] that we can maximize transport capacity (which is defined as the product of transmission rate and distance) by separately optimizing a rate function and then solving for the NA-BOLD solution.

In this paper, we find an analytical solution for the optimal node-activation function, provided a sufficient number of radios are active. In Section 2 we give some details of the system model. In Section 3.1 we present the constrained optimization problem, which was originally described in [9, 8]. In Section 3.2, we reformulate the functional optimization problem as a related measure optimization problem. We investigate the properties of the solution to the measure-theoretic problem, which are then used to derive the optimal measure. In Section 3.3, we show how to map between the optimal measure and the original optimization problem and provide results that compare the performance of the NA-BOLD approaches and conventional approaches to radio activation. Finally, we provide our conclusions in Section 5.

2. System Model. We consider a geographic transmission scheme with a transmitter communicating to multiple neighboring radios over a wireless channel that is subjected to exponential path loss and fading. We assume the amplitude of the random channel fading gain is modeled to vary according to the Nakagami-m distribution [18]. For convenience, we consider that the channel varies slowly enough that the fading amplitude remains constant during a transmission and that the channel gains at different radios are independent. Let H_i denote the power gain of the fading (the square of the fading amplitude) at receiver i. Then $\{H_i\}$ are independent and identically distributed Gamma random variable with distribution function

$$F_H(h) = \frac{1}{\Gamma(m)} \int_0^{mh} t^{m-1} \exp(-t)\, dt, \;\; h > 0 \tag{2.1}$$

where we take $\mathbb{E}[H_i] = 1$ to provide unit average power gain. In (2.1), $\Gamma(m) = \int_0^\infty t^{m-1} \exp(-t)\, dt$, and the parameter $m = (\mathbb{E}[H_i^2] - 1)^{-1},\ m \geq \frac{1}{2}$. The latter is commonly referred to as the *fading figure*, which can be varied to model different fading conditions in wireless links.

We assume that radios are distributed according to a two-dimensional homogeneous Poisson point process with rate β_0 radios per unit area. Thus for radios located in some annulus $[R_1, R_2]$ measured from the source, the distance from the source to radio i, denoted X_i has density

$$f_X(x) = \frac{2x}{R_2^2 - R_1^2}, \quad 0 \leq R_1 < x \leq R_2 < \infty. \tag{2.2}$$

Then the (normalized) signal-to-noise ratio (SNR) at receiver i is

$$\gamma_i = H_i X_i^{-n}, \tag{2.3}$$

where n denotes the path-loss attenuation factor (typically $n \geq 1.5$). A message is successfully received at a radio if the SNR exceeds a threshold ρ that is the same at all the receivers.

3. NA-BOLD : Node-Activation Based on Link Distance. As previously mentioned in Section 1, we are interested in optimizing the way that radios activate to maximize the distance achieved by a geographic transmission under a constraint on the expected number of radios that activate. We assume that the transmitter has no knowledge of the neighboring radios, but that the radios know their distance from the transmitter. We note that as the distance from the transmitter increases, the mean of the SNR decreases, and so the probability of the radio successfully receiving a message decreases. Thus, if receivers close to the transmitter are activated, they have a high probability of receiving the message correctly, but the message makes little progress from the transmitter. On the other hand, if receivers far from the transmitter activate, the message can travel further if it is successfully received; however the probability of the message being successfully received is low. Thus, the optimal way to activate radios may balance among these two factors.

Since the receivers are randomly distributed, we let a receiver determine whether to activate according to a *node-activation function* that depends on the distance of that radio from the transmitter. Thus, our approach is called NA-BOLD (Node-Activation Based-on-Link-Distance). Let $\psi(x)$ be the node-activation function, which is the conditional probability that a radio should activate given that it is at distance x from the transmitter. To successfully receive a message, a radio most activate and the received SNR at that radio must exceed the threshold ρ. Let V_i be the distance to a successful receiver. Then

$$V_i = \begin{cases} X_i, & \gamma_i > \rho \quad \cap \quad U_i < \psi(X_i) \\ 0, & \text{otherwise}, \end{cases} \tag{3.1}$$

where U_i is a Uniform random variable on $[0, 1)$.

Let $Y_i = \sqrt[n]{(H_i \rho^{-1})}$, and denote the distribution and the complementary distribution function of Y by $F(y)$ and $G(y)$ respectively, where $G(y) = 1 - F(y)$. Using (2.1), (2.3) and (3.1), we have for $v > 0$,

$$\begin{aligned} F_V(v) = 1 - P(V_i > v) &= 1 - P(X_i > v,\ X_i < Y_i,\ U_i < \psi(X_i)) \\ &= 1 - \int_v^\infty \psi(x) G(x) f_X(x) dx. \end{aligned} \tag{3.2}$$

3.1. Optimal Node-Activation Function. Consider first transmission to radios in some finite area A, which contains $\aleph$ radios (both awake and asleep). Then the distance to the farthest successful receiver inside this region is

$$V_{\max} = \begin{cases} \max\Big\{V_1, V_2 \dots V_\aleph\Big\}, & \aleph = 1, 2 \dots \\ 0, & \aleph = 0. \end{cases} \tag{3.3}$$

Let $F_{V_{\max}}$ denote the distribution of $V_{\max}$. Then, conditioned on $\aleph = N$, $F_{V_{\max}}(t|\aleph = N) = [F_V(t)]^N$. Since $\aleph$ is a Poisson random variable with mean $\beta_0 A$, the distribution of $V_{\max}$,

$$F_{V_{\max}}(t) = \exp\Big(\beta_0 A \left(F_V\left(t\right) - 1\right)\Big). \tag{3.4}$$

Let K denote the number of radios that activate to attempt to receive a message transmission. Then our optimization problem is

$$\hat{\psi} = \arg\max_{\psi}\ \mathbb{E}\left[V_{\max}\right] \tag{3.5}$$

$$\text{subject to:} \quad \begin{cases} \mathbb{E}\left[K\right] = \mu,\ \mu \text{ given} \\ 0 < \psi \le 1, \end{cases}$$

where $\mathbb{E}[\cdot]$ is used to denote expected value of a random variable. As $V_{\max}$ is non-negative, we can rewrite its expected value, $\mathbb{E}\left[V_{\max}\right]$ as

$$\mathbb{E}\left[V_{\max}\right] = \int_0^\infty \left[1 - \exp\Big(\beta_0 A \left(F_V\left(t\right) - 1\right)\Big)\right]\ dt. \tag{3.6}$$

In what follows, we show that there is an outer radius R_2 such that no radios will be activated beyond R_2 for any finite μ and β_0. Then substituting (3.6) in (3.5), and using (3.2), we can express our optimization problem as

$$\hat{\psi} = \arg\max_{\psi} \left\{ \int_0^{R_2} \left[1 - \exp\left(-\beta_0 A \int_t^{R_2} \psi(x) G(x) f_X(x) dx \right) dt \right] \right\} \tag{3.7}$$

$$\text{subject to:} \quad \begin{cases} \beta_0 A \int_0^{R_2} \psi(x) f_X(x)\ dx = \mu, \ \mu \text{ given} \\ 0 < \psi \leq 1. \end{cases}$$

We have presented a numerical method of approximating $\hat{\psi}$ in [8]. In this paper, we find an analytical solution for $\hat{\psi}$ provided that β_0 is sufficiently large. It is simple to see that the objective function in (3.7) is a monotonically non-decreasing function of β_0 for fixed ψ. In Section 3.2, we reformulate the optimization problem in (3.7) as a problem to find an optimal measure that removes the explicit dependence on β_0 and f_X. In Section 3.3, we show how to calculate the optimal node-activation function from the optimal measure, and we give an expression for the minimum radio density β_0 for which this is possible.

3.2. Optimal Measure. Let us consider a similar optimization problem to (3.7),

$$A(\lambda) = \max_{\lambda} \left\{ \int_0^{\infty} \left[1 - \exp\left(- \int_t^{\infty} G(x) \lambda(dx) \right) \right] dt \right\} \tag{3.8}$$

over all measures λ on $[0, \infty]$ and G is a continuous function on $[0, \infty)$ such that

$$\begin{aligned} \lambda(1) = \lambda(0, \infty) = \mu, \quad \mu \text{ given} \\ \lim_{x \to \infty} x G(x) = 0. \end{aligned} \tag{3.9}$$

We wish to obtain the measure λ that maximizes $A(\lambda)$ as in (3.8). In Section 3.3, we show how the solution for $\lambda(dx)$ in (3.9) can be used to find the solution to (3.7). We start with the continuity of $A(\lambda)$.

LEMMA 3.1. *$A(\lambda)$ is continuous: If $\lambda_n \to \lambda$ weakly [16] then, $A(\lambda_n) \to A(\lambda)$.*

Proof. Now $1 - \exp(-x) \leq x$ so that

$$\begin{aligned} \int_R^{\infty} \left[1 - \exp\left(- \int_t^{\infty} G(x) \lambda_n(dx) \right) \right] dt &\leq \int_R^{\infty} \left(\int_t^{\infty} G(x) \lambda_n(dx) \right) dt \\ &= \int_R^{\infty} (x - R)\, G(x) \lambda_n(dx) \\ &\leq \sup_{x \geq R} (x G(x))\, \lambda_n(R, \infty) \leq \sup_{x \geq R} (x G(x))\, \mu \leq \epsilon \end{aligned}$$

using (3.9). Since G is continuous and $\lambda_n \to \lambda$ weakly,

$$\int_t^{\infty} G(x) \lambda_n(dx) \to \int_t^{\infty} G(x) \lambda(dx)$$

except perhaps for countably mainly t's. By boundedness, we get

$$\int_0^{R} \left[1 - \exp\left(- \int_t^{\infty} G(x) \lambda_n(dx) \right) \right] dt \to \int_0^{R} \left[1 - \exp\left(- \int_t^{\infty} G(x) \lambda(dx) \right) \right] dt.$$

This completes the proof. ∎

We show that λ satisfying (3.8) exists uniquely with the help of the following theorem.

THEOREM 3.2. *There is a unique λ with $\lambda(1) = \mu$ maximizing (3.8).*

Proof. $A(\lambda)$ is strictly concave on the closed convex set $\{\lambda : \lambda(1) = \mu\}$ (closed with respect to weak topology). The continuity was proved in Lemma (3.1). We now prove the existence. Let α be such that

$$\alpha = \sup\{A(\lambda) : \lambda(1) = \mu\}.$$

Also let λ_n be such that,

$$\lambda_n(1) = \mu$$
$$\lim_{n\to\infty} A(\lambda_n) = \alpha.$$

Claim $\{\lambda_n\}$ above is tight [16]. To see this, we start with some simple inequalities. If $0 < a, b < 1$ then $(1 - ab) - (1 - a) = a(1 - b) \le (1 - b)$.
Taking

$$a = \exp\left(-\int_t^R G(x)\lambda(dx)\right) \quad \text{and} \quad b = \exp\left(-\int_R^\infty G(x)\lambda(dx)\right),$$

we get

$$\left[1 - \exp\left(-\int_t^\infty G(x)\lambda(dx)\right)\right] - \left[1 - \exp\left(-\int_t^R G(x)\lambda(dx)\right)\right] \le 1 - \exp\left(-\int_R^\infty G(x)\lambda(dx)\right) \le \int_R^\infty G(x)\lambda(dx). \tag{3.10}$$

Integrating (3.10) from 0 to R, we get

$$\int_0^R \left[1 - \exp\left(-\int_t^\infty G(x)\lambda(dx)\right)\right] dt \le \int_0^R \left[1 - \exp\left(-\int_t^R G(x)\lambda(dx)\right)\right] dt + R\int_R^\infty G(x)\lambda(dx). \tag{3.11}$$

Thus for any $\lambda, \lambda(1) = \mu$, from (3.11),

$$\begin{aligned}
A(\lambda) &= \int_0^\infty \left[1 - \exp\left(-\int_t^\infty G(x)\lambda(dx)\right)\right] dt \\
&= \int_0^R \left[1 - \exp\left(-\int_t^\infty G(x)\lambda(dx)\right)\right] dt + \int_R^\infty \left[1 - \exp\left(-\int_t^\infty G(x)\lambda(dx)\right)\right] dt \\
&\le \int_0^R \left[1 - \exp\left(-\int_t^R G(x)\lambda(dx)\right)\right] dt + R\int_R^\infty G(x)\lambda(dx) + \int_R^\infty dt \int_t^\infty G(x)\lambda(dx) \\
&\le \int_0^R \left[1 - \exp\left(-\int_t^R G(x)\lambda(dx)\right)\right] dt + \int_R^\infty xG(x)\lambda(dx). \qquad (3.12)
\end{aligned}$$

Also,
$$\int_0^R \left[1-\exp\left(-\frac{\mu}{\lambda(0,R)}\int_t^R G(x)\lambda(dx)\right)\right]dt - \int_0^R \left[1-\exp\left(-\int_t^R G(x)\lambda(dx)\right)\right]dt$$
$$= \int_0^R \left[\exp\left(-\int_t^R G(x)\lambda(dx)\right) - \exp\left(-\frac{\mu}{\lambda(0,R)}\int_t^R G(x)\lambda(dx)\right)\right]dt$$
$$\geq \int_0^R \left(\frac{\mu}{\lambda(0,R)}-1\right)\left(\int_t^R G(x)\lambda(dx)\right)\left(\exp\left(-\frac{\mu}{\lambda(0,R)}\int_t^R G(x)\lambda(dx)\right)\right)dt. \quad (3.13)$$

In (3.13) we have used the following inequality. If $Y > X$, then we have the following inequality:

$$\exp(-X) - \exp(-Y) \geq (Y-X)\exp(-Y).$$

Also note that

$$\frac{\mu}{\lambda(0,R)}\int_t^R G(x)\lambda(dx) \leq \mu \sup_x G(x).$$

Using this in (3.13), we get

$$\int_0^R \left[1-\exp\left(-\frac{\mu}{\lambda(0,R)}\int_t^R G(x)\lambda(dx)\right)\right]dt - \int_0^R \left[1-\exp\left(-\int_t^R G(x)\lambda(dx)\right)\right]dt$$
$$\geq \left[\frac{\mu}{\lambda(0,R)}-1\right]\exp\left(-\mu\sup_x G(x)\right)\int_0^R xG(x)\lambda(dx). \quad (3.14)$$

Now we are ready to prove the tightness of $\{\lambda_n\}$. For any λ with $\lambda(1)=\mu$ and any R such that $\lambda(0,R)\neq 0$, the measure $\lambda_R = \frac{\mu}{\lambda(0,R)}\mathbf{1}_{[0,R]}\lambda$ satisfies $\lambda_R(1)=\mu$. Suppose λ is such that $A(\lambda)+\epsilon\alpha \geq \alpha$. Then, using (3.12) we get

$$A(\lambda_R) \leq \alpha \leq A(\lambda)+\epsilon\alpha \leq \int_0^R \left[1-\exp\left(-\int_t^R G(x)\lambda(dx)\right)\right]dt + \int_R^\infty xG(x)\lambda(dx) + \epsilon\alpha. \quad (3.15)$$

Now using (3.14) and noting that $A(\lambda_R)$ is the left most term in (3.14), we get from (3.15),

$$\left[\frac{\mu}{\lambda(0,R)}-1\right]\exp\left(-\mu\|G\|_\infty\right)\int_0^R xG(x)\lambda(dx) \leq \int_R^\infty xG(x)\lambda(dx) + \epsilon\alpha. \quad (3.16)$$

Observe that (3.15) holds for any λ such that $A(\lambda)+\epsilon\alpha \geq \alpha$ and any R such that $\lambda(0,R)\neq 0$. Further for any λ, we have,

$$\int_0^R \left[1-\exp\left(-\int_t^R G(x)\lambda(dx)\right)\right]dt \leq \int_0^R dt \int_t^R G(x)\lambda(dx) = \int_0^R xG(x)\lambda(dx)$$

Therefore, using (3.12), we find that for any λ,

$$A(\lambda) - \int_R^\infty xG(x)\lambda(dx) \leq \int_0^R xG(x)\lambda(dx). \quad (3.17)$$

Since

$$\int_R^\infty xG(x)\lambda(dx) \le \left(\sup_{x\ge R}(xG(x))\right)\mu,$$

we can find R_ϵ so that $\forall\lambda$,

$$\int_{R_\epsilon}^\infty xG(x)\lambda(dx) \le \epsilon\alpha.$$

Therefore, if λ is such that $A(\lambda)+\alpha\epsilon \ge \alpha$, we get from (3.17) with $R = R_\epsilon$,

$$(1-\epsilon)\alpha - \epsilon\alpha \le \int_0^{R_\epsilon} xG(x)\lambda(dx).$$

Using this in (3.16), we find for any λ such that $A(\lambda)+\epsilon\alpha \ge \alpha$ and R_ϵ chosen as above,

$$\frac{\mu}{\lambda(0,R_\epsilon)} - 1 \le \frac{2\epsilon\alpha\exp(\mu\|G\|_\infty)}{(1-2\epsilon)\alpha}.$$

This proves tightness. □ We have proved above that $\exists\lambda_\infty$ (unique, of course) such that

$$A(\lambda_\infty) = \sup_\lambda\{A(\lambda) : \lambda(1) = \mu\}.$$

We now investigate the properties of λ_∞. For convenience of notation, we write λ instead of λ_∞.

THEOREM 3.3. *Let λ satisfy $\lambda(1) = \mu, A(\lambda) = \alpha$. Then λ is supported in an interval $[R_1, R_2], 0 < R_1 < R_2 < \infty$.*

Proof. If $\lambda(0,R) > 0$, then $\lambda_R = \dfrac{\mu}{\lambda(0,R)}\mathbf{1}_{[0,R]}\lambda$ is a candidate in (3.8).

$$\begin{aligned}\int_0^R\left[1-\exp\left(-\int_t^R G(x)\lambda_R(dx)\right)\right]dt &= A(\lambda_R) \le \alpha = A(\lambda)\\ &\le \int_0^R\left[1-\exp\left(-\int_t^R G(x)\lambda(dx)\right)\right]dt + \int_R^\infty xG(x)\lambda(dx)\end{aligned} \tag{3.18}$$

The last inequality follows from (3.12). We rewrite (3.18) as

$$\int_0^R\left[\exp\left(-\int_t^R G(x)\lambda(dx)\right) - \exp\left(-\int_t^R G(x)\lambda_R(dx)\right)\right]dt \le \int_R^\infty xG(x)\lambda(dx),$$

and using (3.14) we get

$$\left[\frac{\mu}{\lambda(0,R)} - 1\right]\exp(-\mu\|G\|_\infty)\int_0^R xG(x)\lambda(dx) \le \int_R^\infty xG(x)\lambda(dx). \tag{3.19}$$

Noting that $\mu = \lambda(1)$, we get from (3.19),

$$\exp(-\mu\|G\|_\infty)\frac{1}{\lambda(0,R)}\int_0^R xG(x)\lambda(dx) \le \frac{1}{\lambda(R,\infty)}\int_R^\infty xG(x)\lambda(dx). \tag{3.20}$$

From (3.9), since $xG(x) \to 0$, as $x \to \infty$, (3.20) cannot hold as $R \to \infty$ unless $\lambda(R,\infty) = 0$ for some R.

Let R_2 be the smallest R such that $\lambda(R,\infty) = 0$. Then for each $R < R_2$,

$$\lambda_R = \frac{\mu}{\lambda(R,R_2)} \mathbf{1}_{[R,R_2]}\lambda$$

is also a candidate. So,

$$A(\lambda_R) \leq \alpha = A(\lambda) = \int_0^{R_2} \left[1 - \exp\left(-\int_t^{R_2} G(x)\lambda_R(dx)\right)\right] dt. \tag{3.21}$$

Hence,

$$\begin{aligned} A(\lambda_R) = &\int_0^{R_2} \left[1 - \exp\left(-\frac{\mu}{\lambda(R,R_2)} \int_t^{R_2} \mathbf{1}_{(R,R_2)} G(x)\lambda(dx)\right)\right] dt \\ = &\int_0^{R} \left[1 - \exp\left(-\frac{\mu}{\lambda(R,R_2)} \int_R^{R_2} G(x)\lambda(dx)\right)\right] dt \\ &+ \int_R^{R_2} \left[1 - \exp\left(-\frac{\mu}{\lambda(R,R_2)} \int_t^{R_2} G(x)\lambda(dx)\right)\right] dt, \end{aligned} \tag{3.22}$$

and

$$A(\lambda) = \int_0^{R} \left[1 - \exp\left(-\int_t^{R_2} G(x)\lambda(dx)\right)\right] dt + \int_R^{R_2} \left[1 - \exp\left(-\int_t^{R_2} G(x)\lambda(dx)\right)\right] dt. \tag{3.23}$$

Using (3.22) and (3.23) in (3.21), we get,

$$\begin{aligned} I_1 &= \int_R^{R_2} \left[\exp\left(-\int_t^{R_2} G(x)\lambda(dx)\right) - \exp\left(-\frac{\mu}{\lambda(R,R_2)} \int_t^{R_2} G(x)\lambda(dx)\right)\right] dt \\ &\leq \int_0^{R} \left[\exp\left(-\frac{\mu}{\lambda(R,R_2)} \int_R^{R_2} G(x)\lambda(dx)\right) - \exp\left(-\int_t^{R_2} G(x)\lambda(dx)\right)\right] dt \\ &= \int_0^{R} \left[\exp\left(-\frac{\mu}{\lambda(R,R_2)} \int_R^{R_2} G(x)\lambda(dx)\right)\right] dt \\ &\quad - \int_0^{R} \left[\exp\left(-\int_R^{R_2} G(x)\lambda(dx)\right) \exp\left(-\int_t^{R} G(x)\lambda(dx)\right)\right] dt \\ &= I_2 - I_3, \end{aligned} \tag{3.24}$$

where we have denoted

$$\begin{aligned} I_2 &= \int_0^{R} \exp\left(-\int_R^{R_2} G(x)\lambda(dx)\right) \left[1 - \exp\left(-\int_t^{R} G(x)\lambda(dx)\right)\right] dt, \text{ and} \\ I_3 &= \int_0^{R} \left[\exp\left(-\int_R^{R_2} G(x)\lambda(dx)\right) - \exp\left(-\frac{\mu}{\lambda(R,R_2)} \int_R^{R_2} G(x)\lambda(dx)\right)\right] dt. \end{aligned}$$

Proceeding as in (3.13), we have,

$$
\begin{aligned}
I_1 &\geq \int_R^{R_2} \left(\frac{\mu}{\lambda(R,R_2)} - 1 \right) \left(\int_t^{R_2} G(x)\lambda(dx) \right) \left(\exp\left(-\frac{\mu}{\lambda(R,R_2)} \int_t^{R_2} G(x)\lambda(dx) \right) \right) dt \\
&\geq \frac{\lambda(0,R)}{\lambda(R,R_2)} \exp\left(-\frac{\mu}{\lambda(R,R_2)} \int_R^{R_2} G d\lambda \right) \int_R^{R_2} dt \int_t^{R_2} G(x)\lambda(dx) \\
&= \frac{\lambda(0,R)}{\lambda(R,R_2)} \exp\left(-\frac{\mu}{\lambda(R,R_2)} \int_R^{R_2} G d\lambda \right) \int_R^{R_2} (x-R)\, G(x)\lambda(dx) \\
&\geq \frac{\lambda(0,R)}{\lambda(R,R_2)} \exp\left(-\mu \|G\|_\infty\right) \int_R^{R_2} (x-R)\, G(x)\lambda(dx), \qquad (3.25)
\end{aligned}
$$

since $\mu = \lambda(1) = \lambda(0, R_2)$. Now,

$$
I_2 \leq \exp\left(-\int_R^{R_2} G(x)\lambda(dx) \right) \int_0^R dt \int_t^R G(x)\lambda(dx) \leq \int_0^R xG(x)\lambda(dx). \qquad (3.26)
$$

From (3.24), (3.25) and (3.26), we get

$$
\frac{\lambda(0,R)}{\lambda(R,R_2)} \exp\left(-\mu\|G\|_\infty\right) \int_R^{R_2} (x-R)\, G(x)\lambda(dx) \leq \int_0^R xG(x)\lambda(dx) \leq \sup_{0<x<R} (xG(x))\, \lambda(0,R). \qquad (3.27)
$$

Since $xG(x) \to 0$ as $x \to 0$, (3.27) cannot hold for small R unless $\lambda(0, R) = 0$. □

THEOREM 3.4. *λ is concentrated on the maxima of the function,*

$$
G(x) \left(\int_0^x \exp\left(-\int_t^\infty G(s)\lambda(ds) \right) dt \right). \qquad (3.28)
$$

Proof. Let ν be any measure such that $\nu(1) = \mu$. Then, for $0 \leq \theta \leq 1$,

$$
\int_0^\infty \left[1 - \exp\left(-\int_t^\infty G(x)\left(\theta\nu(dx) + (1-\theta)\lambda(dx)\right) \right) \right] dt
$$

attains its maximum at $\theta = 0$. So its derivative at $\theta = 0$ is less than or equal to zero. I.e.,

$$
\int_0^\infty \left[G(x) \int_0^x \exp\left(-\int_t^\infty G(s)\lambda(ds) \right) dt \right] (\nu(dx) - \lambda(dx)) \leq 0.
$$

In other words,

$$
\int_0^\infty \tilde{G}(s)\nu(ds) \leq \int_0^\infty \tilde{G}(s)\lambda(ds), \quad \forall \nu : \nu(1) = \mu \qquad (3.29)
$$

where

$$
\tilde{G}(s) = G(s) \int_0^s \exp\left(-\int_t^\infty G(x)\lambda(dx) \right) dt.
$$

Now (3.29) is valid for every ν such that $\nu(1) = \mu$. Taking $\nu = \mu\delta y$, we have

$$\tilde{G}(y) \leq \frac{M}{\mu}, \quad M = \int_0^\infty \tilde{G}(s)\lambda(ds). \tag{3.30}$$

Since $\lambda(1) = \mu$ and

$$\int \tilde{G}(s)\lambda(ds) = M,$$

we further deduce from (3.30) that,

$$\tilde{G}(s) = \frac{M}{\mu} = \sup_y \tilde{G}(y), \quad \lambda \textbf{ a.e.} \tag{3.31}$$

In other words, λ is concentrated on the maxima of $\tilde{G}(y)$. This completes the proof. □

Let us now assume that the support of λ is an interval of the form $[R_1, R_2]$. Using (3.31), we obtain,

$$G(x) \int_0^x \exp\left(-\int_t^\infty G(s)\lambda(ds)\right) dt = \frac{M}{\mu} = C \quad \text{(Say)} \quad , R_1 \leq x \leq R_2 \tag{3.32}$$

λ **a.e.** and hence by continuity and the assumption that the support of λ is $[R_1, R_2]$. Assuming $\frac{1}{G}$ is differentiable, we get from (3.32),

$$\exp\left(-\int_x^\infty G(s)\lambda(ds)\right) = C\frac{d}{dx}\left[\frac{1}{G(x)}\right], \quad R_1 \leq x \leq R_2. \tag{3.33}$$

Assuming that $\log\left(\frac{d}{dx}\left[\frac{1}{G(x)}\right]\right)$ is differentiable, from (3.33) we obtain

$$G(x)\lambda(dx) = \frac{d}{dx}\log\left(\frac{d}{dx}\left[\frac{1}{G(x)}\right]\right).$$

In other words,

$$\lambda(dx) = \frac{1}{G(x)}\frac{d}{dx}\log\left(\frac{d}{dx}\left[\frac{1}{G(x)}\right]\right), \quad R_1 \leq x \leq R_2. \tag{3.34}$$

This determines λ in $[R_1, R_2]$. We still need to find the expressions for R_1, R_2 and C. Substituting $x = R_1$ in (3.32) we get,

$$G(R_1)\int_0^{R_1} \exp\left(-\int_t^\infty G(s)\lambda(ds)\right) dt = C$$

and since λ is concentrated on $[R_1, R_2]$, we thus have,

$$G(R_1)R_1 \exp\left(-\int_{R_1}^{R_2} G(s)\lambda(ds)\right) = C. \tag{3.35}$$

Substituting $x = R_1$ in (3.33), we get

$$\exp\left(-\int_{R_1}^{R_2} G(s)\lambda(ds)\right) = C\frac{d}{dx}\left[\frac{1}{G(x)}\right]_{x=R_1}. \tag{3.36}$$

Using (3.36) in (3.35) we get,

$$
\begin{aligned}
G(R_1)R_1\frac{d}{dx}\left[\frac{1}{G(x)}\right]_{x=R_1} &= 1\\
\text{or,}\qquad R_1G'(R_1)+G(R_1) &= 0. \qquad (3.37)
\end{aligned}
$$

We can solve (3.37) for R_1. We also know that

$$\int_{R_1}^{R_2}\lambda(dx)=\mu$$

and so from (3.34),

$$\int_{R_1}^{R_2}\frac{1}{G(x)}\frac{d}{dx}\left(\log\frac{d}{dx}\left[\frac{1}{G(x)}\right]\right)=\mu. \qquad (3.38)$$

We can solve (3.38) for R_2. Using (3.34) in (3.33), we get,

$$\exp\left(-\int_x^{R_2}\frac{d}{dt}\log\left(\frac{d}{dt}\left[\frac{1}{G(t)}\right]\right)dt\right)=C\frac{d}{dx}\left[\frac{1}{G(x)}\right],\quad R_1\le x\le R_2.$$

The above equation can be further simplified into

$$
\begin{aligned}
\exp\left(\log\left(\frac{d}{dx}\left[\frac{1}{G(x)}\right]\right)-\left[\log\left(\frac{d}{dx}\left[\frac{1}{G(x)}\right]\right)\right]_{x=R_2}\right) &= C\frac{d}{dx}\left[\frac{1}{G(x)}\right]\\
\text{or,}\qquad \frac{1}{C} &= \left[\frac{d}{dx}\left[\frac{1}{G(x)}\right]\right]_{x=R_2}. \qquad (3.39)
\end{aligned}
$$

Thus, we have obtained the expressions for R_1, R_2 and C.

We now specify the conditions so that R_1, R_2 exist uniquely. In (3.34), we require that λ is a positive measure in $[R_1, R_2]$. (3.34) shows that this is only possible if

$$\frac{d}{dx}\left[\frac{1}{G(x)}\right]$$

is increasing in $[R_1, R_2]$, i.e. $1/G$ is convex in $[R_1, R_2]$:

Lemma 3.5. *A measure λ given in (3.34) is positive iff $\frac{1}{G}$ is convex in $[R_1, R_2]$.*

Lemma 3.6. *Suppose G is positive, G' is continuous on $[0,\infty)$,*

$$\lim_{x\to\infty}xG(x)=0,\ \text{and}\ \int_0^\infty G(x)dx<\infty. \qquad (3.40)$$

Then the equation,

$$RG'(R)+G(R)=0 \qquad (3.41)$$

has solutions. If in addition, $\frac{1}{G}$ is convex, then (3.41) has a unique solution.

Proof. Integration by parts gives,

$$\int_0^\infty G(x)dx=-\int_0^\infty xG'(x)dx$$

i.e.

$$\int_0^\infty [G(x) + xG'(x)]\, dx = 0.$$

Therefore, $G(x) + xG'(x)$ must assume both positive and negative values. By continuity, (3.41) has solutions.

Dividing by G^2, we find (3.41) is equivalent to

$$x\frac{G'(x)}{(G(x))^2} + \frac{1}{G(x)} = 0$$

$$\text{or,} \qquad \frac{1}{G(x)} - x\frac{d}{dx}\left[\frac{1}{G(x)}\right] = 0.$$

Now,

$$\frac{d}{dx}\left[\frac{1}{G(x)} - x\frac{d}{dx}\left(\frac{1}{G(x)}\right)\right] = -x\frac{d^2}{dx^2}\left(\frac{1}{G(x)}\right) < 0$$

because $1/G$ is convex. Thus the function

$$\frac{1}{G(x)} - x\frac{d}{dx}\left[\frac{1}{G(x)}\right]$$

is strictly decreasing. So it can have at most one zero.

Now that we have shown the unique existence of R_1, we need to show that R_2 exists uniquely, i.e. (3.38) has a unique solution for each μ, or that,

$$\int_0^\infty \frac{1}{G(x)}\frac{d}{dx}\left(\log\frac{d}{dx}\left[\frac{1}{G(x)}\right]\right) dx = \infty. \tag{3.42}$$

Since $\frac{1}{G}$ is convex and positive, and $G(y) \to 0$ as $y \to \infty$.

$$\int_y^\theta \frac{1}{G(x)}\frac{d}{dx}\left(\log\frac{d}{dx}\left[\frac{1}{G(x)}\right]\right) dx \geq \frac{1}{G(y)}\left[\log\frac{d}{dx}\left(\frac{1}{G(x)}\right)\right]_y^\theta$$

for any θ and $y < \theta$. Since $\frac{d}{dx}\left[\frac{1}{G(x)}\right]$ is increasing, (3.42) is proved. We have thus shown that R_2 exists uniquely. ☐ We now have,

THEOREM 3.7. *Suppose* $\int_0^\infty G(x)dx < \infty, G, G'$ *continuous and* $\frac{1}{G}$ *convex. Then* R_1, R_2 *satisfying* (3.37) *and* (3.38) *exist uniquely. Therefore, the measure* λ *in* (3.34) *satisfies* (3.33) *and also satisfies Theorem 3.4.*

Proof. Only the last two assertions need be established. Call the function in Theorem 3.4 $\tilde{G}(x)$:

$$\tilde{G}(x) = G(x)\left(\int_0^x \exp\left(-\int_{R_1}^\infty G(s)\lambda(ds)\right)\right) dt. \tag{3.43}$$

Then, using (3.36) in (3.43), for $0 \leq x \leq R_1$,

$$\begin{aligned}\tilde{G}(x) &= G(x)\left(\int_0^x \exp\left(-\int_{R_1}^\infty G(s)\lambda(ds)\right)\right) dt \\ &= xG(x)C\left[\frac{d}{d\theta}\left(\frac{1}{G(\theta)}\right)\right]_{\theta=R_1} \\ &= CxG(x)\frac{1}{R_1 G(R_1)}.\end{aligned} \tag{3.44}$$

From (3.33) and (3.37), for $R_1 \le x \le R_2$,

$$\begin{aligned}\tilde{G}(x) &= G(x)\left(\int_0^{R_1} \exp\left(-\int_t^\infty G(s)\lambda(ds)\right) dt + \int_{R_1}^{x} \exp\left(-\int_t^\infty G(s)\lambda(ds)\right) dt\right)\\ &= G(x)\left[CR_1\left\{\frac{d}{d\theta}\left(\frac{1}{G(\theta)}\right)\right\}_{\theta=R_1} + \frac{C}{G(x)} - \frac{C}{G(R_1)}\right]\\ &\equiv C. \end{aligned} \tag{3.45}$$

Finally for $x \ge R_2$,

$$\begin{aligned}\tilde{G}(x) &= G(x)\left[\int_0^{R_1} \exp\left(-\int_t^\infty G(s)\lambda(ds)\right) dt + \int_{R_1}^{R_2} \exp\left(-\int_t^\infty G(s)\lambda(ds)\right) dt\right.\\ &\qquad \left. + \int_{R_2}^{x} \exp\left(-\int_{R_1}^\infty G(s)\lambda(ds)\right) dt\right]\\ &= G(x)\left[CR_1\left\{\frac{d}{d\theta}\left(\frac{1}{G(\theta)}\right)\right\}_{\theta=R_1} + C\left\{\frac{1}{G(R_2)} - \frac{1}{G(R_1)}\right\} + x - R_2\right],\\ &= G(x)\left[\frac{C}{G(R_2)} + x - R_2\right]. \end{aligned} \tag{3.46}$$

(3.45) shows that λ is concentrated on the set where $\tilde{G} = C$. To complete the proof, we must show that $C = \max_{x\ge 0} \tilde{G}(x)$.

To see this, recall that $\lim_{x\to\infty} xG(x) = 0$. From (3.46) and (3.44) we see that $\tilde{G}(x)$ tends to zero as $x \to 0$ and $x \to \infty$. At any maxima of $\tilde{G}$, $\tilde{G}'$ must vanish. Using the fact that R_1 is the unique solution to (3.37) we see that $\tilde{G}'$ cannot vanish in the open intervals $(0, R_1)$ and (R_2, ∞). This concludes the proof. □ Finally we need to show that λ defined by (3.34) indeed maximizes $A(\lambda)$ defined in (3.8):

Theorem 3.8. *For every measure ν on $[0, \infty)$, with $\nu(1) = \mu$, $A(\nu) \le A(\lambda)$.*

Proof. The proof is contained in Theorem 3.4, but we spell out the details.

Let ν be any other measure with $\nu(1) = \mu$. The function,

$$B(\theta) = \int_0^\infty \left[1 - \exp\left(-\int_t^\infty G(x)\left(\nu\theta(dx) + (1-\theta)\lambda(dx)\right)\right)\right] dt$$

is strictly concave in $0 \le \theta \le 1$. Its derivative at $\theta = 0$ is,

$$\int_0^\infty \left[G(x)\int_0^x \exp\left(-\int_t^\infty G(s)\lambda(ds)\right) dt\right] (\nu(dx) - \lambda(dx)) \le 0$$

because

$$\begin{aligned}\int_0^\infty \left[G(x)\int_0^x \exp\left(-\int_t^\infty G(s)\lambda(ds)\right) dt\right] \nu(dx) &= \int_0^\infty \tilde{G}(x)\nu(dx)\\ &\le \left(\max_x \tilde{G}(x)\right)\nu(1)\\ &= C\mu\\ &= \int_0^\infty \tilde{G}(x)\lambda(dx)\end{aligned}$$

as proved in Theorem 3.7. □

3.3. Computing Optimal Node-Activation Function from Optimal Measure. The optimal measure λ is absolutely continuous with respect to the Lebesgue measure and has density

$$\lambda(dx) = \frac{1}{G(x)} \frac{d}{dx} \log\left(\frac{d}{dx}\left[\frac{1}{G(x)}\right]\right), \quad R_1 \leq x \leq R_2 \tag{3.47}$$

$$\text{and satisfies} \quad \int_{R_1}^{R_2} \lambda(x)dx = \mu. \tag{3.48}$$

Here, R_1 can be obtained by solving (3.37), and R_2 can be obtained by solving (3.38). With the help of (2.2), (3.7) and substituting the activation area $A = \pi(R_2^2 - R_1^2)$ in (3.48),

$$\beta_0 \pi \int_{R_1}^{R_2} \frac{\lambda(x)}{\beta_0 \pi 2x} 2x dx = \mu. \tag{3.49}$$

We can thus write the optimal node-activation function from Section 3.1 as

$$\hat{\psi}(x) = \frac{\lambda(x)}{2\beta_0 \pi x}, \tag{3.50}$$

where β_0 needs to be sufficiently large to guarantee $0 < \psi \leq 1$. This is satisfied if

$$\beta_0 \geq \frac{1}{2\pi} \max_{R_1 < x \leq R_2} \left(\frac{\lambda(x)}{x}\right). \tag{3.51}$$

(3.51) gives us an expression for the minimum radio density required to ensure $0 < \psi \leq 1$.

4. Results. We have evaluated the performance of our NA-BOLD approach in the Nakagami-m fading channel. We provide results for $m = 1$, commonly known as the Rayleigh fading channel model. The SNR threshold at every receiver is assumed to be unity, i.e. $\rho = 1$. The path-loss attenuation factor is $n = 4$, and the radio density is $\beta_0 = 10$ radios per unit area.

We compare the performance of the optimal NA-BOLD scheme (NA-BOLD(O)) discussed in this paper with several other NA-BOLD schemes. In [8], we have already presented a computational method of approximating the optimal NA-BOLD (referred as the NA-BOLD(O) scheme in [8]). We refer to the computational method as NA-BOLD(C) in this paper. In [9, 8], we also consider a sub-optimal NA-BOLD scheme (NA-BOLD(S)) scheme that is based on solving for the optimal distribution of distances for a fixed number of radios. We then use this to determine the node-activation function that achieves that distribution and satisfies the constraint on the expected number of active radios. We also compare our NA-BOLD schemes with an approach that turns on all of the radios out to some fixed radius around the transmitter. We call this as the DISC scheme. The radius is chosen so that the expected number of radios that activate in the DISC scheme is the same as for the NA-BOLD schemes.

The results in Fig. 4.1 show the expected value of the transmission distance to the farthest receiver to successfully receive the message as a function of the expected number of radios that activate. The results show that the NA-BOLD schemes significantly outperform the DISC scheme. Further, we find that all the NA-BOLD schemes show extremely similar performance (within a margin of 1% accuracy).

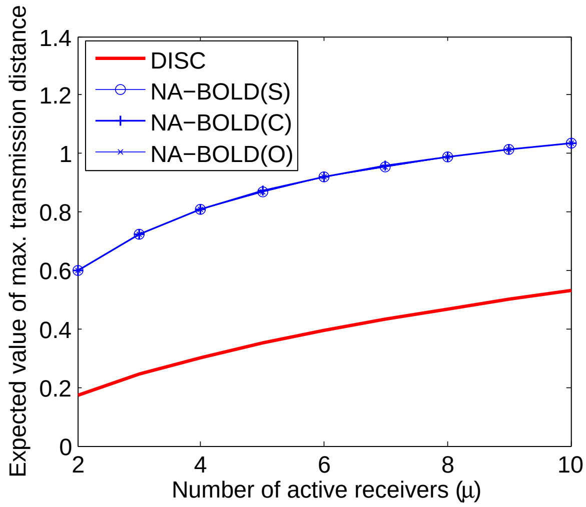

FIG. 4.1. *Expected value of the maximum transmission distance* $\mathbb{E}[V_{\max}]$ *vs. the expected number of active radios* μ.

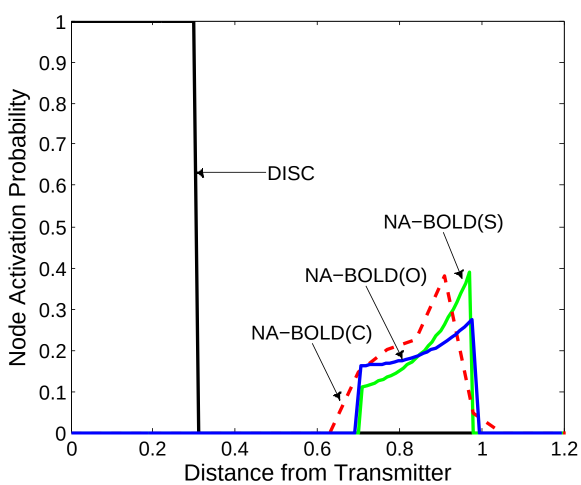

FIG. 4.2. *Node-activation probability for* $\beta_0 = 10$ *radios per unit area, with expected number of active radios,* $\mu = 3$.

We have also plotted the node-activation probability for these schemes in Fig. 4.2 for $\mu = 3$. The NA-BOLD approaches do not turn on radios close to the transmitter, and the probability of a radio activating increases with distance from the transmitter. Since a radio (active or inactive) in the activation region has a higher probability of being closer to the outer radius R_2 than the inner radius R_1 (cf. (2.2)), there is an even higher probability that the activated radios will be concentrated close to R_2. Thus, the NA-BOLD schemes are more aggressive than DISC, as they turn on more radios that are located far away from the transmitter.

5. Conclusion. In this paper, we consider the problem of maximizing the expected transmission distance for geographic transmissions in fading channels, under a constraint on the expected number of radios that activate to try to recover the transmission. Different from previous work, we follow a measure-theoretic framework to solve our optimization problem. We investigate the properties of the optimal measure and derive conditions on when it exists uniquely. We derive the optimal measure and map it to the optimal distance-based node activation function that receivers can use to determine whether to activate to try to receive a transmission. We present results to show that the NA-BOLD schemes offer significantly better performance than a scheme that just turns on all of a radio's neighbors out to some radius that achieves the same expected number of active radios. Our optimization problem is similar to the Monge-Kantorovich optimal mass transport formulation [15, 11, 23], which is an area for future investigation.

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