

Chapter 7

Multiuser Detection

We have discussed a simple method of MAI suppression in Chapter 6. The idea of MAI suppression stems from the single-user detection philosophy, in which we treat signals from other users as interference. In this chapter, we will introduce another way, namely multiuser detection, to tackle the near-far problem. In the paradigm of multiuser detection, we jointly detect data signals from all the users. There is not MAI since signals from all users are treated as the desired signals. However, one should note that this does not mean that we do not need to design spreading sequences with “good” correlation properties for the users. For example, if two users use exactly the same sequence spreading sequence and their transmissions are synchronous, there is not way we can reliably resolve the data signals of the two users from the received signal even if we detect them jointly. On the other hand, with properly designed sequences, the near-far problem is solved implicitly by performing multiuser detection.

Theoretically, receivers based on multiuser detection usually outperform, but are usually more complex than, receivers based on single-user detection. The applicability of multiuser receivers depends on system design issues, such as the security of joint detection, the implementation complexity, and the availability of information required to perform multiuser detection. For example, let us consider a typical wireless cellular system. It would be difficult to employ multiuser receivers at the mobile units for forward-link transmission because of the limitation on the implementation complexity and the availability of information about other users. However, multiuser detection could be a viable choice in the base-station for reverse-link transmission.

In this chapter, we will introduce some common multiuser detection techniques. We will start with the optimal multiuser receiver and then discuss some suboptimal but simpler receivers. As we mentioned in Chapter 6, we will also discuss some single-user MAI suppression receivers which happen to be specializations of the suboptimal multiuser receivers described in this chapter.

7.1 Maximum Likelihood Sequence (MLS) receiver

First, let us spell out the mathematical model of DS-CDMA system we use throughout the chapter. Basically, we consider the same asynchronous DS-CDMA model in Section 6.1 with the following simplifications:

1. Short sequences are employed and for notational convenience, we use $a_k(t)$ to represent one period of the spreading signal, i.e.,

$$a_k(t) = \sum_{l=0}^{N-1} a_l^{(k)} \psi(t - lT_c). \quad (7.1)$$

2. A finite number of symbols are transmitted for each user. For the k th user, we define its symbol vector as

$$\mathbf{b}_k = [b_{-M}^{(k)}, b_{-M+1}^{(k)}, \dots, b_0^{(k)}, \dots, b_{M-1}^{(k)}, b_M^{(k)}]^T. \quad (7.2)$$

Hence, the received signal can be written as

$$r(t) = s(t, \mathbf{b}) + n(t), \quad (7.3)$$

where $n(t)$ is AWGN with power spectral density N_0 ,

$$s(t, \mathbf{b}) = \sum_{k=1}^K \sqrt{2P_k} e^{j\phi_k} \sum_{i=-M}^M b_i^{(k)} a_k(t - iT - \tau_k), \quad (7.4)$$

and

$$\mathbf{b} = [\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_K]. \quad (7.5)$$

The form of (7.3) actually embodies the idea of multiuser detection — all the user's signals are treated as desired signals. With this in mind, we note that (7.3) is exactly the same as (1.1), the simple M-ary signaling in an AWGN channel, except that we have $2^{K(2M+1)}$ possible "symbols" and the symbol

duration is much longer than T in the multiuser case here. The whole development in Section 1.1 applies and the ML receiver ¹ decides \mathbf{b} , all the bits of the users, which maximizes the following correlation metric,

$$c(\mathbf{b}, r(t)) = \int_{-\infty}^{\infty} r(t)s^*(t, \mathbf{b})dt - \frac{1}{2} \int_{-\infty}^{\infty} |s(t, \mathbf{b})|^2 dt. \quad (7.6)$$

Let us define

$$z_i^{(k)} = \int_{-\infty}^{\infty} r(t)a_k^*(t - iT - \tau_k)dt \quad (7.7)$$

and

$$\begin{aligned} \rho_{i,j}^{(k,l)} &= \int_{-\infty}^{\infty} a_k(t - iT - \tau_k)a_l^*(t - jT - \tau_l)dt \\ &= \int_0^T a_k(t - (i-j)T - (\tau_k - \tau_l))a_l^*(t)dt \\ &= R_\psi(T_c - \delta^{(k,l)})C_{k,l}(\gamma_{i,j}^{(k,l)}) + \hat{R}_\psi(T_c - \delta^{(k,l)})C_{k,l}(\gamma_{i,j}^{(k,l)} + 1), \end{aligned} \quad (7.8)$$

where

$$\gamma_{i,j}^{(k,l)} = (i-j)N + \left\lfloor \frac{\tau_k - \tau_l}{T_c} \right\rfloor, \quad (7.9)$$

$$\delta^{(k,l)} = \tau_k - \tau_l - \gamma_{i,j}^{(k,l)}T_c + (i-j)N. \quad (7.10)$$

Then the correlation metric in (7.6) can be written as

$$c(\mathbf{b}, r(t)) = \sum_{k=1}^K \sqrt{2P_k} e^{-j\phi_k} \sum_{i=-M}^M b_i^{(k)*} z_i^{(k)} - \sum_{k=1}^K \sum_{l=1}^K \sqrt{P_k P_l} e^{j(\phi_k - \phi_l)} \sum_{i=-M}^M \sum_{j=-M}^M b_i^{(k)} b_j^{(l)*} \rho_{i,j}^{(k,l)}. \quad (7.11)$$

From (7.11), we see that the statistics $z_i^{(k)}$ for $-M \leq i \leq M$ and $1 \leq k \leq K$ are sufficient for the detection of \mathbf{b} . As a result, the MLS receiver consists of a branch of matched filters which are matched to the spreading signals of the users as shown in Figure 7.1.

We note that a brute-force maximization of $c(\mathbf{b}, r(t))$ would require a complexity of $O(2^{K(2M+1)})$ which is by no means practical. It has been shown [1] that the MLS receiver can be implemented by the Viterbi algorithm with a complexity of $O(2^{K-1})$ in a feed-forward fashion. This makes the MLS receiver practical when the number of users in the system is small, say, less than 10. Unfortunately, this is seldom the case for a practical CDMA system. Therefore the use of the MLS receiver is limited.

¹In this case, the ML receiver is called the ML sequence receiver since the whole sequences of data symbols of the users are detected.

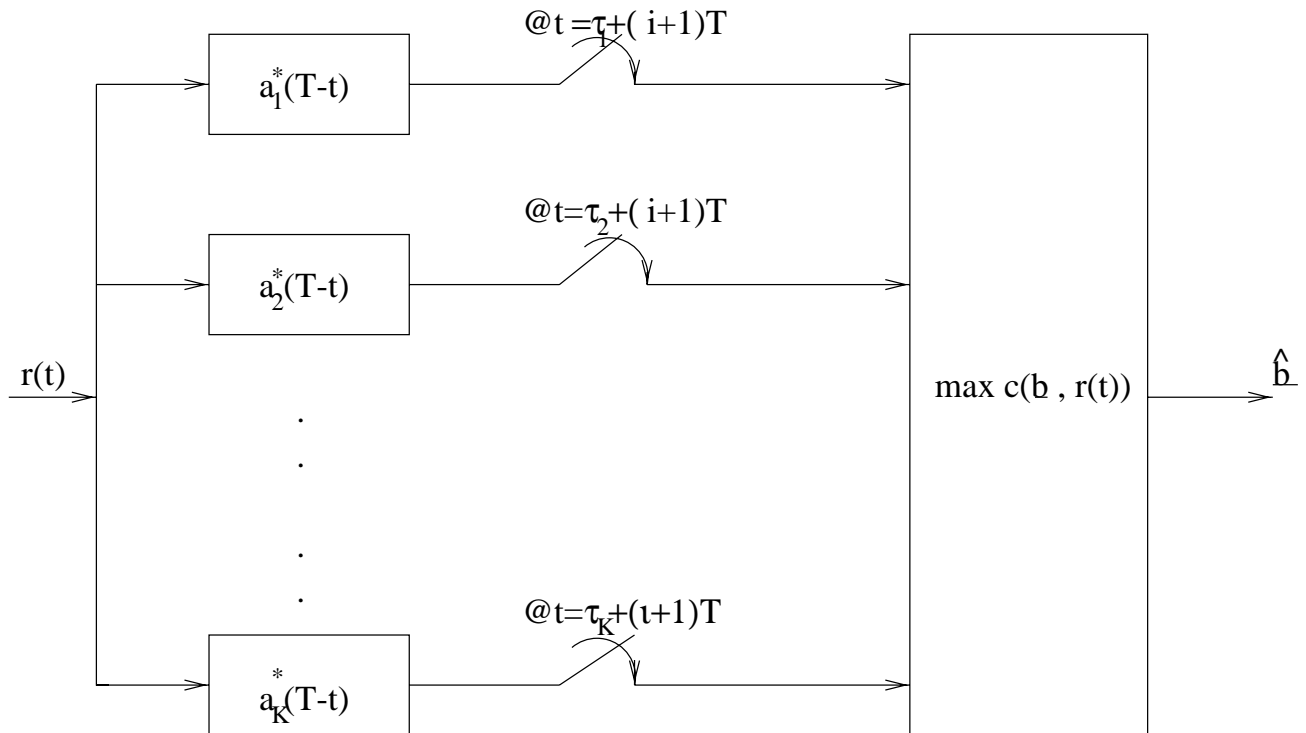


Figure 7.1: Maximum likelihood sequence receiver

Moreover, we note that in order to implement the MLS receiver, besides knowing all the spreading sequences, we need to estimate the delays, carrier phases as well as the receiver power of all the users. Usually, this estimation is very hard to achieve and thus further limits the usefulness of the MLS receiver. Nevertheless, the MLS receiver is important since it gives us a benchmark to gauge the performances of other sub-optimal receivers. It is shown in [1] that the MLS receiver is near-far resistant. We also note that the MLS receiver minimizes the joint error probability of all the bits of all the users. Another possible approach is to minimize the error probability of each individual bit. Optimization under this approach leads to another multiuser receiver [1] which is even more complex than the MLS receiver discussed here.

More to come ...

7.2 References

- [1] S. Verdú, “Minimum Probability of Error for Asynchronous Gaussian Multiple-Access Channels,” *IEEE Trans. Inform. Theory*, vol. 32, pp. 85–96, Jan. 1986.
- [2] S. Verdú, *Multiuser Detection*, Cambridge University Press, 1998.
- [3] A. Duel-Hallen, J. Holtzman, and Z. Zvonar, “Multiuser Detection for CDMA Systems,” *IEEE Personal Commun.*, vol. 2, no. 2, pp. 46–58, Apr. 1995.