High Capacity Fixed Wireless Access Systems with Antenna Arrays

Tat M. Lok
Department of Information Engineering
The Chinese University of Hong Kong
Shatin, Hong Kong

and

Tan F. Wong
Department of Electrical and Computer Engineering
University of Florida
Gainesville, Florida 32611, U. S. A.

Abstract- In this paper, we consider fixed wireless access (FWA) systems with antenna arrays. We consider both the downlink and the uplink. Multiple antennas are used at the transmitters, as well as at the receivers. We demonstrate that with antenna arrays, the user capacity of a system can be dramatically increased, allowing multiple users to use the same narrowband spectrum simultaneously. In general, it may be difficult to determine the optimal transmission vectors. We consider a simple algorithm that often yields desirable solutions. We also consider the special case of co-located users. The communication system then reduces to a point-to-point communication link where the optimal transmission vectors can be determined under some conditions.

I. INTRODUCTION

Fixed wireless access (FWA) can be an attractive alternative to traditional wireline technologies to provide high-speed access to homes and small businesses [1]. FWA systems enjoy the advantages of rapid deployment, small investment, and low maintenance and upgrade costs. However, the scarcity of the usable frequency spectrum may limit the number of users that can be supported by an FWA system.

Antenna arrays have been shown to provide dramatic improvements in the performance of different communication systems (e.g., [1]-[2], and their references). Since the physical constraints are less demanding in FWA systems than in mobile communications systems, the application of antenna arrays in FWA systems can be particularly attractive. In this paper, we consider FWA systems with antenna arrays. We demonstrate how antenna arrays can be used to dramatically increase the user capacity of an FWA system. We consider both the downlink and the uplink. Interesting similarities are observed on the downlink and the uplink of a time division multiplexed (TDD) system.

In Section II, we investigate the downlink. We present the system model and provide a necessary condition and a sufficient condition for a set of users requiring different qualities of service to be supported by the system. We also consider a simple iterative algorithm to determine the desirable transmission vectors. Simulation examples are provided to illustrate the results. In Section III, we consider the uplink and provide the relevant results. In Section IV, we consider the special case where the users are co-located and share the same antenna array. The system then reduces to a point-to-point communication link. In this case, the optimal transmission vectors can be found under some conditions. Conclusions are presented in Section V.

II. DOWNLINK

We first consider the downlink (from the central station to the users). For simplicity, we assume a synchronous communication system although the results can be generalized to asynchronous systems.

A. System model

We assume that there are $K$ users with access to a central station in the system. During each symbol interval, the central station sends a data symbol $s_k$ with zero mean and unit variance to the $k$th user, for $1 \leq k \leq K$. The symbol $s_k$ is to be transmitted via an antenna array of $N$ elements, and is multiplied by a (possibly different) complex gain at each element. Therefore, the transmitted signal for the $k$th user, in discrete time representation, is the vector $s_k e_k$, where $e_k$ is the $N$-dimensional transmission vector for the $k$th user. The total transmitted signal is given by the vector

$$ t = \sum_{k=1}^{K} s_k e_k. \tag{1} $$

We assume that the signal from each transmitting antenna to each receiving antenna undergoes independent flat fading. For each user, an antenna array of $M$ elements is used to capture the signal. Without loss of generality, we consider the first user. The received signal is the $M$-dimensional vector given by

$$ r_1 = n_1 + G_1 \sum_{k=1}^{K} s_k e_k \tag{2} $$

where $n_1$ is an $M$-dimensional vector representing the contribution of the additive white Gaussian noise (AWGN), and $G_1$ is the $M \times N$ dimensional channel gain matrix for the $k$th user. The $(i, j)$ element of $G_1$ represents the complex gain of the signal from the $j$th antenna of the central station to the $i$th antenna of the $k$th user. We assume that the first $M$ columns of $G_k$, for $k = 1, \ldots, K$, are linearly independent if $M \leq N$, and the first $N$ rows of $G_k$, for $k = 1, \ldots, K$, are linearly independent if $N \leq M$. (These assumptions guarantee that the antennas are not redundant.)

We consider simple correlation receivers. The receiver for the first user combines the components of $r_1$ with a weight vector $w_1$ to give a decision statistic $z_1 = w_1^T r_1$. We determine the weight
vector that maximizes the signal-to-noise ratio (SNR) defined by

$$\text{SNR}_1 = \frac{|w_1^H G_1 c_1|^2}{\mathbb{E} \left[ \left| w_1^H (n_1 + G_1 \sum_{k=2}^K s_k c_k) \right|^2 \right]} = \frac{|w_1^H G_1 c_1|^2}{w_1^H R_1 w_1}. \quad (3)$$

The noise-plus-interference correlation matrix $R_1$ observed by the first user is given by

$$R_1 = \sigma^2 I + G_1 \left( \sum_{k=2}^K c_k c_k^H \right) G_1^H \quad (4)$$

where $\sigma^2$ is the variance of the AWGN. It can be shown [3] that the optimal weight vector is given by

$$w_1 = R_1^{-1} G_1 c_1. \quad (5)$$

With the optimal weight vector, the SNR at the output of the receiver for the first user is given by

$$\text{SNR}_1 = c_1^H G_1 R_1^{-1} G_1 c_1. \quad (6)$$

Similar results are obtained for other users.

We express the quality of service requirement for each user in terms of a target SNR. We assume that the $k$th user requires a target SNR of $\gamma_k$. We would like to determine transmission vectors $c_k$ for $k = 1, 2, \ldots, K$, so that the target SNR's for all users can be achieved and the total transmission power is minimized. Hence, we seek the solution to the optimization problem:

$$\min \sum_{k=1}^K |c_k|^2 \quad (7)$$

subject to

$$\text{SNR}_k \geq \gamma_k \quad \text{for} \quad k = 1, \ldots, K. \quad (8)$$

B. Necessary condition and sufficient condition

Obviously, there may not be a solution to the optimization problem if there are too many users or many users require high SNR's. Proposition 1 provides a sufficient condition for a set of users to be supported.

Proposition 1: A set of $K$ users with target SNR's $\gamma_1, \ldots, \gamma_K$ can be supported if

$$\sum_{k=1}^K \frac{\gamma_k}{1 + \gamma_k} < \min(M, N). \quad (9)$$

Proof: We first consider the case where $M = N$. For $K \leq N$, we can pick orthogonal vectors $c_1, \ldots, c_K$ as the transmission vectors. We then pick weight vectors $w_k$ for $k = 1, \ldots, K$, so that $w_k$ is orthogonal to $G_k c_j$ for $j \neq k$, but not orthogonal to $G_k c_k$. There would be no interference from other users. With sufficiently large power, the target SNR's can be achieved.

We consider the case where $K > N$. It can be shown that the target SNR constraint (8) for the $k$th user using the optimal weight vector is equivalent to the following constraint:

$$c_k^H G_k R_k^{-1} G_k c_k \geq \zeta_k \quad (10)$$

where

$$R_{T,k} = \sigma^2 I + G_k \left( \sum_{j=1}^K c_j c_j^H \right) G_k^H \quad (11)$$

and

$$\zeta_k = \frac{\gamma_k}{1 + \gamma_k}. \quad (12)$$

The left side of (10) can be re-expressed as

$$c_k^H \left( \sigma^2 G_k^{-1} (G_k^H)^{-1} + \sum_{j=1}^K c_j c_j^H \right)^{-1} c_k. \quad (13)$$

We determine the largest eigenvalues of $G_k^{-1} (G_k^H)^{-1}$ for $k = 1, \ldots, K$. We pick a constant $\lambda_{\max}$ which dominates these eigenvalues. Notice that for any positive definite matrices $R_a$ and $R_b$ and any vector $c$,

$$c^H R_a^{-1} c \geq c^H (R_a + R_b)^{-1} c. \quad (14)$$

Therefore, (13) is larger than the following expression:

$$c_k^H \left( \sigma^2 \lambda_{\max} I + \sum_{j=1}^K c_j c_j^H \right)^{-1} c_k \quad (15)$$

The proposition is proved for $M = N$ if we can find $c_k$ so that the last expression is larger than or equal to $\zeta_k$ for $k = 1, \ldots, K$. Define the $N \times K$ matrix $C$ so that the $k$th column of the $C$ is $c_k$. Equivalently, we need to prove that

$$\text{diag} \left[ C^H (C C^H + \sigma^2 \lambda_{\max} I)^{-1} C \right] \geq [\zeta_1, \ldots, \zeta_K] \quad (16)$$

where the operator $\text{diag} \{ \cdot \}$ takes the diagonal of a matrix to form a row vector, and the inequality is interpreted as element-by-element comparisons. It is shown in Appendix III of [4] that for $K > N$, we can obtain an appropriate $N \times K$ matrix. The rows of the matrix are orthonormal and the columns have norm squares $e_1, \ldots, e_K$ with the following properties:

- $\gamma_k \leq e_k \leq 1$ for $k = 1, \ldots, K$,
- $\sum_{k=1}^K e_k = N$.

The resulting matrix is scaled by a constant $g$ to give the desired $C$ matrix with the property that

$$\text{diag} \left[ C^H (C C^H + \sigma^2 \lambda_{\max} I)^{-1} C \right] = \frac{g^2}{g^2 + \sigma^2 \lambda_{\max}} [e_1, \ldots, e_K]. \quad (17)$$

For sufficiently large $g$, (16) and, hence, (10) can be satisfied.

For the case of $M > N$, we can shut off the last $M - N$ antennas at each receiver by setting the relevant coefficients of the weight vectors to zero, and use the result for the case of $N$ transmitting antennas and $N$ receiving antennas.

For the case of $N > M$, we can shut off the last $N - M$ antennas at the transmitter by setting the relevant coefficients of the transmission vectors to zero, and use the result for the case of $M$ transmitting antennas and $M$ receiving antennas.

Depending on the channel gain matrices, a set of users may be supported even if the sufficient condition is not satisfied. However, the following condition is necessary:
Proposition 2: If a set of $K$ users with target SNR's $\gamma_1, \ldots, \gamma_K$ can be supported, then the target SNR's satisfy the condition
\[
\sum_{k=1}^{K} \frac{\gamma_k}{1 + \gamma_k} < \max(M, N).
\]  
(18)

Proof: The condition is always satisfied for $K \leq \max(M, N)$. We consider the case where $K > \max(M, N)$. If the users are supported, the target SNR's are achieved, which implies
\[
\zeta_k \leq c_k^H G_k^H R_k^{-1} G_k c_k.
\]  
(19)

Notice that the optimal SNR's of the users are at least as good as before, with the addition of extra antennas (because we can set the relevant coefficients of the extra antennas to zero). Equivalently, the right side of (19) can only increase with the addition of extra antennas. Therefore, to determine the desired upper bound on $\zeta_k$, we can consider the case where $M = N$. (We can increase the smaller one so they are equal.) In this case, the right side can be expressed as
\[
c_k^H \left( \sigma^2 G_k^{-1} (G_k^H)^{-1} + \sum_{j=1}^{K} c_j c_j^H \right)^{-1} c_k.
\]  
(20)

We determine the smallest eigenvalue of $G_k^{-1} (G_k^H)^{-1}$ for $k = 1, \ldots, K$. We pick a positive constant $\lambda_{\min}$ which is smaller than these eigenvalues. Notice that the last expression is smaller than the following expression:
\[
c_k^H \left( \sigma^2 \lambda_{\min} I + \sum_{j=1}^{K} c_j c_j^H \right)^{-1} c_k
\]  
(21)

In matrix form, we have
\[
[\zeta_1, \ldots, \zeta_K] \leq \text{diag} \left[ C^H (C C^H + \sigma^2 \lambda_{\min} I)^{-1} C \right].
\]  
(22)

The sum of the elements on both sides have to satisfy the corresponding inequality.

\[
\sum_{k=1}^{K} \zeta_k \leq \text{tr} \left[ C^H (C C^H + \sigma^2 \lambda_{\min} I)^{-1} C \right]
\]  
(23)

\[
= \text{tr} \left[ C C^H (C C^H + \sigma^2 \lambda_{\min} I)^{-1} \right]
\]  
(24)

\[
= \text{tr} \left[ \Lambda Q^{-1} Q (\Lambda + \sigma^2 \lambda_{\min} I)^{-1} Q^{-1} \right]
\]  
(25)

\[
= \text{tr} \left[ \Lambda (\Lambda + \sigma^2 \lambda_{\min} I)^{-1} \right]
\]  
(26)

\[
= \sum_{n=1}^{N} \frac{\lambda_n}{\lambda_n + \sigma^2 \lambda_{\min}} < \sum_{n=1}^{N} \frac{\lambda_n}{\lambda_n + \sigma^2 \lambda_{\min}}
\]  
(27)

where we have performed spectral factorization for $C C^H$ into $Q \Lambda Q^{-1}$. Since $C C^H$ is Hermitian and positive semi-definite, $\lambda_n \geq 0$ for $n = 1, \ldots, N$ and $Q$ is unitary.

The results imply that with antenna arrays, more users can be supported by the system. For example, 10 users, each with a target SNR of 6 dB, can be supported simultaneously in a narrowband system, if each user, as well as the central station, uses an antenna array of 8 elements. Notice that without using antenna arrays, only one user can be supported at a time in this system.

C. Simple iterative algorithm

In general, it is difficult to obtain a closed form solution for the optimization problem. However, various numerical algorithms can be used to tackle the problem. In particular, we consider a simple iterative algorithm to determine appropriate transmission vectors.

At each iteration, $c_k$, for $k = 1, \ldots, K$, are updated iteratively as follows:

1. Update the direction of $c_k$

\[
c_k \leftarrow c_k + \mu G_k^H R_k^{-1} G_k c_k
\]  
(24)

where $\mu$ is a constant.

2. Update the power of $c_k$

\[
c_k \leftarrow \sqrt{\frac{\gamma_k}{c_k G_k^H R_k^{-1} G_k c_k}} c_k
\]  
(25)

Simulations show that the algorithm converges in a small number of iterations. Fig. 1 shows the typical performance of the algorithm. We consider a system with 10 simultaneous users. Each user, as well as the central station, uses an antenna array of 8 elements. We see that each user achieves a target SNR of 6 dB in a small number of iterations. Fig. 2 shows a case where the sufficient condition is violated while the necessary condition is satisfied. The system supports 6 simultaneous users, each with a target SNR of 6 dB. The central station uses an antenna array of 8 elements while the users use antenna arrays of 4 elements.

III. Uplink

We consider the uplink. Each user transmits with an antenna array of $M$ elements to a central station using an antenna array of $N$ elements for reception. The $k$th user transmitted signal, in discrete time representation, is the vector $\delta_k \xi_k$, where $\delta_k$ is the symbol to be transmitted and $\xi_k$ is the $M$-dimensional transmission vector.

\footnote{1 We use the same notation with the addition of a /up to distinguish an uplink quantity from a downlink quantity.}
For a TDD system, the channel gain matrix on the uplink for the $k$th user can be represented by $G_k^H$. The received signal at the central station is the $N$-dimensional vector given by

$$f_1 = \tilde{n}_1 + \sum_{k=1}^{K} \delta_k G_k^H \epsilon_k$$ (26)

where $\tilde{n}_1$ is an $N$-dimensional vector representing the contribution of the AWGN.

We consider simple decentralized correlation receivers. Without loss of generality, we consider the first user. The receiver for the first user combines the components of $f_1$ with a weight vector $\tilde{w}_1$ to give a decision statistic $\tilde{\epsilon}_1 = \tilde{w}_1^H f_1$. We determine the weight vector that maximizes the SNR defined by

$$\text{SNR}_1 = \frac{\mathbb{E}[|\tilde{w}_1^H G_1^H \tilde{\epsilon}_1|^2]}{\mathbb{E}[|\tilde{w}_1^H G_1^H \tilde{\epsilon}_1|^2]}$$

$$= \frac{\tilde{w}_1^H \tilde{R}_1 \tilde{w}_1}{\sum_{k=2}^{K} \delta_k G_k^H \epsilon_k}$$ (27)

It can be shown that the optimal weight vector is given by

$$\tilde{w}_1 = \tilde{R}_1^{-1} G_1^H \epsilon_1$$ (28)

The noise-plus-interference correlation matrix $\tilde{R}_1$ for the first user is given by

$$\tilde{R}_1 = \sigma^2 I + \sum_{k=2}^{K} \delta_k G_k^H \epsilon_k G_k$$ (29)

and is quite different from the one for the downlink. With the optimal weight vector, the SNR at the output of the receiver for the first user is given by

$$\text{SNR}_1 = \tilde{\epsilon}_1^H G_1 \tilde{R}_1^{-1} G_1^H \tilde{\epsilon}_1$$ (30)

We again assume that the $k$th user requires a target SNR of $\gamma_k$.

We determine transmission vectors $\epsilon_k$, for $k = 1, 2, \ldots, K$, so that the target SNR’s for all users can be achieved and the total transmission power is minimized. Hence, we seek the solution to the optimization problem:

$$\min \sum_{k=1}^{K} |\tilde{\epsilon}_k|^2$$ (31)

subject to

$$\text{SNR}_k \geq \gamma_k \text{ for } k = 1, \ldots, K$$ (32)

Propositions 3 and 4 show that the same necessary condition and the same sufficient condition hold for a set of users to be supported by the system.

Proposition 3: A set of $K$ users with target SNR’s $\gamma_1, \ldots, \gamma_k$ can be supported if

$$\sum_{k=1}^{K} \frac{\gamma_k}{1 + \gamma_k} < \min(M, N).$$ (33)

Proof: We first consider the case where $M = N$. For $K \leq N$, we can pick transmission vectors $\epsilon_1, \ldots, \epsilon_K$ so that $G_k^H \epsilon_k$ are orthogonal vectors. There would be no interference from other users if $\tilde{w}_k$ are properly chosen. With sufficiently large power, the target SNR’s can be achieved.

We consider the case where $K > N$. It can be shown that the target SNR constraint for the $k$th user using the optimal weight vector is equivalent to the following constraint:

$$\tilde{\epsilon}_k^H G_k \tilde{R}_T^{-1} G_k^H \tilde{\epsilon}_k \geq \zeta_k$$ (34)

where

$$\tilde{R}_T = \sigma^2 I + \sum_{j=1}^{K} G_j^H \epsilon_j \tilde{\epsilon}_j^H G_j$$ (35)

and

$$\zeta_k = \frac{\gamma_k}{1 + \gamma_k}.$$ (36)

Let $d_k = G_k^H \epsilon_k$, and define the $N \times K$ matrix $D$ so that the $k$th column of the $D$ is $d_k$. Then the constraints in (34) can be rewritten as

$$\text{diag}[D^H(DD^H + \sigma^2 I)^{-1}D] \geq [\zeta_1, \ldots, \zeta_K]$$ (37)

where the operator $\text{diag}[\cdot]$ takes the diagonal of a matrix to form a row vector, and the inequality is interpreted as element-by-element comparisons. Similar to the construction in Proposition 1, the desired transmission vectors can be constructed. The cases for $M > N$ and $N > M$ are handled the same way as in Proposition 1.

Proposition 4: If a set of $K$ users with target SNR’s $\gamma_1, \ldots, \gamma_k$ can be supported, then the target SNR’s satisfy the condition

$$\sum_{k=1}^{K} \frac{\gamma_k}{1 + \gamma_k} < \max(M, N).$$ (38)

Proof: If the users are supported, the target SNR’s are achieved.

$$\zeta_k \leq \tilde{\epsilon}_k^H G_k \tilde{R}_T^{-1} G_k^H \tilde{\epsilon}_k$$ (39)
subject to
\[ \sum_{i=1}^{K} \frac{\beta_i}{1 + \beta_i} = \frac{K - \gamma}{1 + \gamma} \]  
(42)

and
\[ 0 \leq \beta_1 \leq \beta_2 \leq \cdots \leq \beta_K \]  
(43)

Construct the $N \times K$ matrix $\tilde{U}$ from the last $K$ columns (keeping the original order) of $U$ and the $K \times K$ unitary matrix $V$ such that the diagonal elements of
\[ V \left[ \begin{array}{ccc} \beta_1 & & \\ \vdots & \ddots & \vdots \\ \beta_K & & \beta_K \end{array} \right] V^H \]  
(44)

are all equal to $\frac{\gamma}{1 + \gamma}$. Then, the columns of
\[ C = \tilde{U} \left[ \begin{array}{ccc} \sqrt{\beta_1 \lambda_{N-K+1}} & & \\ & \ddots & \\ & & \sqrt{\beta_K \lambda_N} \end{array} \right] V^H \]  
(45)

are the optimal choices of the transmission vectors that require the minimum total power.

Proof: See the proof of Theorem 2 in [5].

Notice that in this case where $M = N$, $K \leq N$, while a simple orthogonal assignment of transmission vectors shows that the target SNR’s can be achieved with suitable choices of weight vectors, Proposition 5 shows that this orthogonal solution is in general sub-optimal.

V. CONCLUSIONS

We have considered fixed wireless access (FWA) systems with antenna arrays. Multiple antennas are used at the transmitters, as well as the receivers. We have investigated both the downlink and the uplink. We have demonstrated that with antenna arrays, the user capacity of a system can be dramatically increased, allowing multiple users to use the same narrowband spectrum simultaneously. We have determined necessary and sufficient conditions for a set of users with given target SNR’s to be supported by a system. In general, it may be difficult to determine the optimal transmission vectors. We provide a simple algorithm that yields desirable solutions. We also consider the special case where the system reduces to a point-to-point communication link. In this case, the optimal transmission vectors can be determined under some conditions.

REFERENCES


