RLS-based Adaptive Multicode CDMA System

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Abstract—A single-user adaptive multicode CDMA system over AWGN channel is presented. The adaptive system involves joint transmitter-receiver adaptation which allows the desired user to achieve a designated signal-to-noise ratio (SNR) with minimum transmission power. We assume that the adaptive transmitter and receiver have no prior knowledge of the spreading sequences of other users. During the training period, receiver weights are adapted based on the RLS algorithm. Information about the MMSE error and weights collected at the receiver is employed to update the spreading sequences, which are fed back to the transmitter for the transmission of the next block of data. Simulation results show that both the transmitter and receiver can converge to the optimal solution. The proposed implementation is also shown to be immune to feedback delay.

Keywords—Multicode CDMA, spreading sequence optimization, RLS algorithm

I. INTRODUCTION

In CDMA communication systems, the near-far problem occurs whereby the weak signal from a distant user is overwhelmed by the strong signal from nearby interference. One of the most popular scheme to combat the near-far problem is transmitter power control. Base station periodically sends information to each mobile to instruct them to adjust their transmission power so that all signals arrive the base station with equal power. Another approach is to use an equalizer, such as the MMSE receiver [1], to reject multiple-access interference (MAI).

While adaptive power control and adaptive MMSE receivers are useful to guarantee a certain SNR at the receiver, they do not explore the full potential of CDMA systems. It is argued in [2] that the SNR achieved by the MMSE receiver depends on the choice of spreading sequences. If the spreading sequences are chosen or adapted suitably together with adaptive power control, MAI can be further suppressed and hence the performance of the system improves. A number of results regarding transmitter optimization are available. The optimization problem of choosing a set of signature sequences with minimum total transmission power such that the target SNR of the users are met over an AWGN channel is solved in [3]. Variants of this optimization problem in a multicarrier setting [4] and a multicode setting [2] are also solved recently. Transmitter adaptation in a multicode CDMA system is discussed in [2], but the adaptive algorithm proposed in [2] assumes that the receiver works with the optimal weights and requires exact knowledge of correlation matrix of received signal. These assumptions limit the application of the algorithm in practical systems. The potential of MAI suppression through joint transmitter-receiver adaptation is discussed in [5], but no indication on how well the joint adaptive technique performs is given.

Based on the theoretical analysis in [2], we present here a practical RLS algorithm based structure for joint transmitter-receiver adaptation in a synchronous multicode CDMA system over AWGN channel. We also compare the performance of this adaptive structure to the optimal solution given in [2]. We consider the problem from the point of view of a single user and assume that all other users do not adapt their spreading sequences. This problem describes the scenario in which a new user with high priority is admitted into the system and is about to adapt its transmission sequences. The objective is to allow the new user to achieve a designated SNR with the minimum transmission power. We assume no prior knowledge of the spreading sequences of other users or the noise floor. During the initial training period, the receiver weights are adapted using the RLS algorithm. Information about the MMSE error and weights collected at the receiver is used to update the spreading sequences. The updated spreading sequences are fed back to the transmitter and are used for the transmission of the next block of data. Hence the performance of the system can be optimized by adapting the transmitter spreading sequences and the receiver weights simultaneously.

The rest of the paper is organized as follows. In Section II, we describe the system model. In Section III, we consider the optimization of the receiver and the transmitter. In Section IV, we propose an RLS-based implementation of the joint adaptive structure. Numerical examples obtained from computer simulation are provided in Section V to verify the performance of our implementation.

II. SYSTEM MODEL

The general structure of the adaptive CDMA system is illustrated in Fig. 1. We assume that there are \( K \) simultaneous users in the system. The \( k \)th user, for \( 1 \leq k \leq K \), generates \( M_k \) streams of data symbols. Altogether, there are \( M = \sum_{k=1}^{K} M_k \) streams of data symbols to be transmitted. The \( m \)th data stream, for \( 1 \leq m \leq M \), is given by \( (b_0^{(m)}, b_1^{(m)}, b_2^{(m)}, \ldots) \). We assume the data symbols \( b_j^{(m)} \) are independent random variables with zero mean and unit variance. The \( k \)th user generates \( M_k \) periodic spreading sequences of period \( N \). The spreading sequence to spread the \( m \)th data stream, for \( 1 \leq m \leq M \), is given by \( (a_0^{(m)}, a_1^{(m)}, \ldots, a_{N-1}^{(m)}) \). Here, we use the notation \( a_m \).
to denote the vector \([a_0^{(m)}, a_1^{(m)}, \ldots, a_{N-1}^{(m)}]^T\) which contains one period of the \(n\)th spreading sequence. The \(n\)th data stream is spread with \(a_n\), and then modulated to the carrier frequency \(\omega_c\) to give the transmitted signal

\[
s_m(t) = \text{Re} \left[ \sum_{t=-\infty}^{\infty} b^{(m)}_{\lfloor t/T_c \rfloor} a_i^{(m)} \psi(t - iT_c)e^{j\omega_c t} \right],
\]

where \(T_c\) is the chip interval, and \(\psi(t)\) is the chip waveform. We assume that \(\psi(t)\) satisfies the Nyquist criterion for zero inter-chip interference, and \(\int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1\). The transmitted signal for the \(k\)th user consists of a sum of \(M_k\) signals of the form in (1).

We now describe the channel model. We consider a synchronous DS-CDMA system in an AWGN channel with power spectral density \(\eta\). Complex baseband representation of the received signal of \(k\)th user is given by

\[
r_k(t) = \sum_{m=1}^{M} e^{j\theta_m} \sum_{t=-\infty}^{\infty} b^{(m)}_{\lfloor t/T_c \rfloor} a_i^{(m)} \psi(t - iT_c) + n(t),
\]

where \(\theta_m\) accounts for the overall phase shift of the \(m\)th signal, and \(n(t)\) represents the AWGN.

We assume that each data stream in the system is demodulated separately by a linear receiver. The received signal is passed through a chip-matched filter and the filter output is sampled every chip interval. We observe continuously \(N\) samples during each symbol interval and feed them into an adaptive FIR filter. During the training period, the weights of linear receiver are adapted every symbol according to the MMSE criterion. The decision statistic \(Z\) for the symbol is obtained as a linear combination of the \(N\) samples weighted by the FIR filter coefficients. Information about the MMSE error and weights is collected to update the spreading sequence. The updated spreading sequence is fed back to the transmitter for the transmission of next block of symbols. The receiver keeps adapting its coefficients and updating the spreading sequence. This adaptation process continues until the receiver output SNR converges.

III. RECEIVER AND TRANSMITTER OPTIMIZATION

In order to provide a measure to the performance of our joint adaptation structure, we address the optimal solution briefly in this section. We focus on the transmission and reception of the first user's signals.

A. Receiver Optimization

Without loss of generality, we consider the first stream of the first user. The \(N\) samples observed at the output of the chip-matched filter are arranged into an \(N\)-dimensional column vector \(z\), which can be expressed as

\[
z = [h_0^{(1)} a_1 + \sum_{m=2}^{M} e^{j\beta_m h_0^{(m)}} a_m + n].
\]

We assume that carrier synchronization has been achieved with the first signal.

The decision statistic \(Z\) is obtained as a linear combination of the \(N\) samples, i.e., \(Z = w^T z\). The weight vector that minimizes the mean square error is

\[
w_1 = R_T^{-1} a_1,
\]

where \(R_T\) is the total correlation matrix observed by the first user which is given by

\[
R_T = \sum_{m=1}^{M} a_m a_m^H + \eta I.
\]

We note that the receiver output SNR is also maximized with this optimal weight vector. The resulting receiver output SNR is given by

\[
\text{SNR}_1 = a_1^H R_T^{-1} a_1,
\]

where the noise-plus-interference correlation matrix \(R_1\) is defined as \(R_1 = \mathbb{E}[n n^H] = R_T - a_1 a_1^H\).

We note that the results above can be applied to any other stream by simply replacing the subscript 1 with the corresponding subscript of that stream.

B. Transmitter Optimization

The goal of the first user is to achieve some target SNR's for all its data streams with the minimum amount of total transmission power. Mathematically, the optimization problem can be expressed as

\[
\min \sum_{m=1}^{M_1} |a_m|^2
\]

subject to

\[
a_m^H R_m^{-1} a_m = \gamma_m
\]

for \(1 \leq m \leq M_1\), where \(R_m = R_T - a_m a_m^H\).

For \(M_1 = 1\), a closed-form solution for this optimization problem can be readily obtained by the method of Lagrange multiplier. The spreading sequence \(a_1\) should be chosen as the eigenvector associated with the smallest eigenvalue, \(\lambda_{\text{min}}\), of
R_{1}. With this choice, the minimal transmission power is given by \(|a_1|^2 = \lambda_{\text{min}} \gamma_1\).

For \(M_1 > 1\), the multistream optimization problem in (7) is much more complex. The optimal solution is discussed in [2]. Here, we just restate the optimization solution. Suppose \(M_1 \leq N\). Let \(R_{1, M_1} = R_T - \sum_{m=1}^{M_1} a_m a_m^H\) be the noise-plus-interference correlation matrix observed by the first user and \(R_{1, M_1} = U\hat{U}^H\) be the spectral factorization of \(R_{1, M_1}\) with eigenvalues \(\lambda_m\) arranged in a descending order in the diagonal matrix \(\Lambda\). Let \(\beta_m\), for \(m = 1, \ldots, M_1\) be the solution of the optimization problem:

\[
\min_{\substack{\beta_m}} \sum_{m=1}^{M_1} \lambda_{N-M_1+m} \beta_m
\]

subject to

\[
0 \leq \beta_1 \leq \beta_2 \leq \cdots \leq \beta_{M_1},
\]

(8)

Construct the \(N \times M_1\) matrix \(\hat{U}\) from the last \(M_1\) columns (keeping the original order) of \(U\) and construct the \(M_1 \times M_1\) unitary matrix \(V\) such that

\[
diag\left[ V \begin{bmatrix} \frac{\lambda_{1+M_1}}{1+\beta_{M_1}} \\ \beta_{M_1}/(1+\beta_{M_1}) \\ \vdots \\ \frac{\lambda_{N-M_1+1}}{1+\beta_{M_1}} \end{bmatrix} V^H \right] = \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_{M_1} \end{bmatrix}
\]

where \(\gamma_m = \frac{\lambda_m}{1+\lambda_m}\). Then,

\[
A_{1, M_1} = \hat{U} \begin{bmatrix} \sqrt{\beta_1 \lambda_{N-M_1+1}} \\ \sqrt{\beta_2} \\ \sqrt{\beta_3} \\ \vdots \\ \sqrt{\beta_{M_1} \lambda_{N-M_1}} \end{bmatrix} V^H
\]

(9)

is a solution to the optimization problem in (7) and the resulting minimum total transmission power is \(\sum_{m=1}^{M_1} \lambda_{N-M_1+m} \beta_m\).

The general solution for the optimization problem in (8) is given in [2]. For the special case in which \(\gamma_m = \gamma\) for \(m = 1, 2, \ldots, M_1\) and

\[
\frac{\lambda_{N-M_1+i}}{\lambda_{N-M_1+m}} < \frac{1 + \gamma}{M_1}
\]

for \(i = 1, \ldots, M_1\), the solution is in simple form:

\[
\beta_i = \frac{1 + \gamma \sum_{m=1}^{M_1} \sqrt{\lambda_{N-M_1+m}}}{M_1 \sqrt{\lambda_{N-M_1+i}}} - 1.
\]

(10)

This optimal choice of the sequence has a simple physical interpretation. Each eigenvector of \(R_{1, M_1}\) represents a "channel" in the CDMA system and the corresponding eigenvalue indicates the degree of "crowdedness" in that channel. To minimize the transmission power, the optimal sequences should, of course, choose the channels that are least congested. More discussion about this optimization problem and solution can be found in [2].

### IV. Joint Receiver and Transmitter Adaptation

To calculate the optimal sequences, the transmitter needs to stay idle until the receiver finishes estimating \(R_{1, M_1}\), solves the complex eigen-problem, and feeds back the optimal sequence. To solve the problem practically, we develop a joint adaptation structure and iterative algorithms to adapt the receiver weights and transmitter spreading sequence. Different from the transmitter adaptation in [2], our implementation does not require perfect knowledge of the total correlation matrix \(R_T\), and does not assume that the receiver works with the optimal weights. The only requirement is an initial training sequence.

The iterative procedure of adapting the receiver weights and transmitter spreading sequence of the \(n\)th signal consists of the following steps:

Step 1: Transmitter transmits the \(j\)th block of training symbols (\(L\) symbols) of the \(n\)th signal with current spreading sequence \(a_m[l]\). (The initial spreading sequence \(a_m[1]\) can be set arbitrarily.)

Step 2: Receiver employs the RLS algorithm to adapt the filter weights using the \(j\)th block of symbols. The filter coefficients are adjusted according to following equations:

\[
e_m(l+1) = b_m(l+1) - w_m^H(l)z_j(l+1),
\]

\[
K_j(l+1) = \frac{R_{T_j}^{-1}(l)z_j(l+1)}{\lambda + z_j^H(l+1)R_{T_j}^{-1}(l)z_j(l+1)}
\]

\[
R_{T_j}^{-1}(l+1) = \frac{1}{\lambda} [R_{T_j}^{-1}(l) - K_j(l+1)z_j^H(l+1)R_{T_j}^{-1}(l)],
\]

\[
w_m(l+1) = w_m(l) + K_j(l+1)e_m(l+1).
\]

The time-averaged squared error for the \(j\)th block of symbols is calculated by

\[
MSE_{e_m} = \frac{1}{L} \sum_{l=1}^{L} e_m^2(l).
\]

(12)

Step 3: The spreading sequence is updated according to

\[
a_m[j+1] = g_{m, j} w_m(l + 1),
\]

(13)

where the coefficient \(g_{m, j}\) is chosen so that

\[
a_m^H[j+1] R_{T_j}^{-1}(l+1) a_m[j+1] = \gamma_m MSE_{e_m}.
\]

(14)

The new spreading sequence \(a_m[j+1]\) is fed back to transmitter for the transmission of the \((j+1)\)th block of the training symbols. The adaptation process repeats from Step 1 until the receiver output SNR and transmission power converge.

There are several points we would like to address here. First, in order to improve the ability to track the inverse correlation matrix \(R_{T_j}^{-1}(l)\) in the adaptive implementation, we use an exponentially weighted RLS algorithm with a forgetting factor \(\lambda\). Usual choices of \(\lambda\) range from 0.8 to 1. The RLS algorithm can converge very fast even when the eigenvalue value spread...
of the correlation matrix is large. Second, the main computation complexity burden of RLS algorithm lies in the updating of the inverse correlation matrix. In this implementation, the same inverse correlation matrix $R_{i,j}^{-1}(l)$ can be used at the receivers of all the streams of the same user, and thus $R_{i,j}^{-1}(l)$ is only calculated once for all the receivers involved. A remaining question is how frequently the spreading sequence should be updated (i.e., what should be a good choice of the symbol block length $L$). A discussion on this is given in next section. Finally, we mention that if continuous updating of the spreading sequence is desired after the initial training period, symbol decisions made by the receiver can be employed to replace the training symbols.

V. SIMULATION ANALYSIS

In this section, we study the performance of proposed adaptive implementation using computer simulations. The performance of the system is measured via the receiver output SNR which is defined as

$$SNR = \frac{\text{instantaneous power of signal}}{\text{average power of interference and noise}}.$$  

(15)

With this definition, the receiver output SNR of the $l$th symbol in the $j$th block of data of the $n$th signal during adaptation is given by

$$SNR_{m,j}(l) = \frac{|w_{m,j}(l)H a_m[j]|^2}{w_{m,j}(l)H R_{m,j} w_{m,j}(l)},$$  

(16)

where $R_{m,j} = \sum_{n=1}^{M} a_n[j]a_n[j]^H + \eta I - a_m[j]a_m[j]^H$. The total transmission power at the $j$th transmitter adaptation step is given by

$$P[j] = \sum_{m=1}^{M_1} |a_m[j]|^2.$$  

(17)

We assume that the spreading factor $N$ is 31. Originally there are $M = 34$ active signal streams in the system. The power spectral density $\eta$ of AWGN satisfies $\eta = \frac{1}{\text{SNR}} = 5$. A new user is admitted into the system. We assume that this new user is going to send $M_1$ streams simultaneously.

First, we study the effect of different choices of the block length $L$. A training sequence of 2000 symbols is sent in blocks with $L = 4, 20$, and 100, respectively. We assume that $M_1 = 1$. The designated SNR is $\gamma = 7$dB. Fig. 2 shows the receiver output SNR and the transmitter transmission power during the adaptation process. With small block length $L$, the SNR reaches the designated SNR very quickly but with large overshoots and oscillations. As $L$ increases, the overshoots and oscillations become weaker, but it takes more training symbols to reach the designated SNR. We also observe that the convergent rate in terms of the number of training symbols needed does not show much difference for different choices of block length. This observation is reasonable. The transmitter spreading sequence is updated more frequently with small $L$. On the other hand, each update with large $L$ is more effective since it employs a better estimate of the correlation matrix. In summary, we can reach the target SNR very quickly through frequent updating with small $L$. However, frequent updating means a waste of bandwidth in the feedback channel. Overshoots also waste the transmission power. With large $L$, we can reduce the waste of feedback channel bandwidth and transmission power, but the output SNR reaches the target more slowly.

Next, we examine the case of multistream adaptation. We set the number of streams of the new user to $M_1 = 3$ and set a uniform target SNR of $\gamma = 7$dB to all of its streams. A training sequence of 2000 symbols is sent with block length $L = 50$. Fig. 3 shows the adaptation process. We can see that the receiver output SNR's of all the streams converge close to the target SNR and the transmission powers approach the minimum transmission power.

In the above simulation, we have assumed that the transmitter can make use of the new spreading sequence immediately after it is updated. This assumption is not valid in practical systems since there is always a delay for the updated spreading sequence to reach back the transmitter. To accommodate this
VI. Conclusion

In this paper, we have proposed and simulated a practical structure for transmitter-receiver adaptation in a synchronous multicode CDMA system. Compared to the general adaptive linear receiver, which only adjusts the receiver weight, our proposed structure is only a slightly more complex. However, both theoretical analysis and computer simulation results show that the system performance can be substantially improved by the proposed joint adaptation technique.

REFERENCES