Multi-dimensional parity-check codes for bursty channels

Tan F. Wong and John M. Shea[1]
Department of Electrical and
Computer Engineering
University of Florida
Gainesville, FL 32611, U.S.A.
e-mail: twong@ece.ufl.edu

Abstract — **We examine a class of multi-dimensional parity-check (MDPC) codes that can be obtained by generalizing the single parity-check code to multiple dimensions geometrically. We demonstrate that the MDPC codes can achieve close-to-capacity performance with a simple iterative decoding algorithm in AWGN as well as bursty channels.**

I. Summary

Let $u_{i_1,i_2,\ldots,i_M}$ be a block of data bits indexed by the set of $M(>1)$ indices $i_1, i_2, \ldots, i_M$. We assume that each of the indices takes values ranging from 1 to A, i.e., there are altogether A^M data bits. This is equivalent to saying that we arrange the data bits into the lattice points of an M-dimensional hypercube of side A. An M-dimensional parity check code is defined as a systematic code in which a codeword consists of the A^M data bits and MA parity bits satisfying

$$p_{m,j} = \sum_{i_1} \cdots \sum_{i_{m-1}} \sum_{i_{m+1}} \cdots \sum_{i_M} u_{i_1,\ldots,i_{m-1},j,i_{m+1}\ldots,i_M}$$

for $m = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, A$. Each sum above ranges over A elements, and additions are modulo-2. Geometrically, the parity bit $p_{m,j}$ checks the data bits on the $(M-1)$-dimensional hyperplane $i_m = j$. Equivalently, the M-dimensional parity-check code can be viewed as a punctured version of the M-time product of the single parity-check code $(A+1, A)$. The code rate of the MDPC is $1/(1+M/A^{M-1})$ and the minimum code weight is $\min(M+1, 4)$. We note that the 2DPC codes have been suggested in [1] and the MDPC codes can be viewed as special cases of the codes considered in [2] with a very regular structure.

Although the MDPC codes have small minimum distances, they can correct a large number of error patterns of larger weights because of their geometric constructions. With suitable interleaving schemes, the MDPC codes are effective for channels with occasional noise bursts. To examine this claim, we employ the simple two-state hidden Markov model to model bursty channels [3]. The system enters State **B** when the channel is having a noise burst. In State **N**, usual AWGN is the only noise. In State **B**, the burst noise is modeled as AWGN with a power spectral density that is B times higher than that of the AWGN in State **N**.

"Soft-in/soft-out" iterative decoding [1] is performed to approximate the maximum *a posteriori* (MAP) decoder. The MDPC code is treated as a parallel concatenation of parity check codes, each defined by the parity-check bits along one dimension and all of the data bits. Simulation results of a number of MDPC codes with block sizes of 10000 and 60000 bits are summarized in the following table:

Code	Code Rate	E_b/N_0 at 10^{-5} **BER**	**Coding gain at** 10^{-5} **BER**	E_b/N_0 **at capacity**
100^2	0.9804	13.7dB	4.15dB	8.4dB
21^3	0.9932	14.8dB	3.05dB	12.0dB
10^4	0.9960	15.5dB	2.35dB	13.1dB
245^2	0.9919	14.1dB	3.75dB	11.5dB
39^3	0.9980	15.45dB	2.4dB	14.2dB
9^5	0.9992	16.6dB	1.25dB	15.4dB

In the simulation, the probabilities of staying in States **B** and **N** are 0.99 and 0.9995, respectively. Moreover, $B =$ 10dB. This represents a case that long noise bursts occur occasionally. Random interleavers of sizes equal to the block size of the MDPC codes are employed. The results are obtained after 10 iterations for all the codes. The decoding process essentially converges after 5 iterations. From the table, with a block size of 10000 bits, the 4DPC code 10^4 can achieve a BER of 10^{-5} within 2.4dB of the capacity limit When the block size goes up to 60000 bits, the 5DPC code 9^5 can achieve a BER of 10^{-5} within 1.2dB of the capacity limit. In an AWGN channel, for instance, the 3DPC code 39^3 can achieve a BER of 10^{-5} at 7.9dB, 0.5dB higher than the capacity limit.

II. Conclusion

MDPC codes are very high-rate systematic codes that have very regular geometric structures. With a simple and fast converging iterative decoding scheme, MDPC codes can achieve close-to-capacity performance with large block sizes in both AWGN and bursty channels. These codes are effective for relatively benign channels with occasional long bursts of errors since only a minimal amount of redundancy is needed. MDPC codes are also be good choices of outer codes for serial concatenated systems with turbo codes as the inner codes [4].

References

[1] J. Hagenauer, E. Offer, and L. Papke, "Iterative decoding of binary block and convoultional codes," *IEEE Trans. Inform. Theory*, vol. 42, pp. 429–445, Mar. 1996.

[2] L. Ping, S. Chan, and K. L. Yeng, "Iterative decoding of multi-dimensional concatenated single parity check codes," in *Proc. IEEE ICC '98*, vol. 1, pp. 131–135, Atlanta, GA, Jun. 1998.

[3] K. Koike and H. Ogiwara, "Application of turbo codes for impulsive noise channel," *IEICE Trans. Fundamentals*, vol. E81-A, no. 10, pp. 2032–2039, Oct. 1998.

[4] J. M. Shea, "Improving the performance of turbo codes through concatenation with rectangular parity check codes," in *Proc. 2001 IEEE Int. Symp. Inform. Theory*, Washington, D.C., June 2001.

[1] John M. Shea was supported by the Office of Naval research under contract number N00014-00-0565.

0-7803-7123-2/01/$10.00 ©2001 IEEE