Minimum-TSC Sequences and Their Application in a CDMA Forward Link

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Abstract

In this paper, two power and sequence allocation schemes are introduced to support users in a CDMA forward link with a uniform target signal-to-interference ratio (SIR). The signal of each user is demodulated using the matched filter receiver. One scheme employs signature sequences that minimize the total squared correlation (TSC) among all unit-energy sequences, and the other employs signature sequences that minimize the extended TSC (ETSC) among all sequences (incorporating user powers). Power efficiencies of these two schemes are studied.

Keywords
Code division multiple access, forward link, total squared correlation, sequence allocation, power control

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I. INTRODUCTION

In CDMA communication systems, the cross-correlations between signature sequences of different users are desired to be small. The total squared correlation (TSC), which indicates the degree of mutual interference among all sequences, is a gauge of performance for a specific set of sequences. When all sequences are of equal energy, the minimum possible value of the TSC is called the Welch bound [1]. The TSC-minimizing sequences are called the Welch-Bound-Equality (WBE) sequences. The sum capacity of a synchronous CDMA system with equal powers is maximized by the WBE sequences [2]. For a synchronous CDMA system with unequal powers, the sum capacity is maximized by the choice of generalized Welch-Bound-Equality (GWBE) sequences [3], which are shown to minimize the extended TSC (ETSC) [4]. Recently, applications of the minimum-TSC sequences in multipath [5] and asynchronous [6] CDMA systems are also studied.

In this paper, we consider the application of the minimum-TSC (WBE) sequences and the minimum-ETSC (GWBE) sequences in a CDMA forward link and study the performance of these two classes of sequences when the matched filter receiver is used. We assume that all users are to be supported with a uniform target signal-to-interference ratio (SIR), and signals from the base-station to different users undergo independent slow flat fading.

The rest of the paper is organized as follows. In Section II, we define the system model and characterize the user capacity of the system. In Section III, we describe the two power and sequence allocation schemes. Power efficiencies of these two schemes are evaluated in Section IV. Conclusions are drawn in Section V.

II. SYSTEM MODEL AND USER CAPACITY

We assume that there are $K$ users in the system. The data stream of the $k$th user, for $1 \leq k \leq K$, is allocated power $p_k$ and is spread with the unit-energy signature sequence $s_k$ of length $N$. We consider forward-link transmission and assume that the signal from the base-station to each user undergoes independent slow flat fading. Signals of all users are detected independently using the matched filter receiver. All users are supported with a uniform target SIR $\gamma$. This implies that the output SIR at the $k$th user’s receiver, for $1 \leq k \leq K$, satisfies

$$ SIR_k = \frac{g_k^2 p_k}{\sum_{i \neq k} g_{ki} p_i |s_k^H s_i|^2 + \eta_k} \geq \gamma, $$

(1)
where $g_k$ denotes the channel gain from the base-station to the $k$th user’s receiver, $\eta_k$ is the power spectral density (PSD) of the thermal noise, and $\tilde{\eta}_k = \frac{\eta_k}{g_k}$ is the effective noise density at the $k$th user’s receiver. The SIR constraints in (1) can be equivalently expressed as

$$p_k \geq e(\gamma) \left( \sum_{l=1}^{K} p_l c_{kl} + \tilde{\eta}_k \right),$$

(2)

where $e(\gamma) = \frac{\gamma}{1+\gamma}$ and $c_{kl} = |s_k^H s_l|^2$. Because $s_k$ is of unit energy, we have $c_{kk} = 1$ and $0 \leq c_{kl} \leq 1$ for $k \neq l$. Writing in matrix form, we have

$$\begin{bmatrix} \frac{1}{e(\gamma)} \mathbf{I} - \mathbf{C} \end{bmatrix} \mathbf{p} \geq \tilde{\eta},$$

(3)

where $\mathbf{p} = [p_1, p_2, \cdots, p_K]^T$, $\tilde{\eta} = [\tilde{\eta}_1, \tilde{\eta}_2, \cdots, \tilde{\eta}_K]^T$ are $K$-dimensional column vectors, and $\mathbf{C} = (c_{kl})$ is a $K \times K$ square symmetric nonnegative matrix. The necessary and sufficient condition for a valid power solution $\mathbf{p}$ ($\mathbf{p} \geq 0, \neq 0$) is that the largest (also called Perron-Frobenius) eigenvalue of $\mathbf{C}$ is smaller than $\frac{1}{e(\gamma)}$ [7]. Follow a derivation similar to the one in [6], one can show that the maximum number of users that can be supported in the system is

$$K < \frac{N}{e(\gamma)}$$

(4)

with proper choices of signature sequences.

### III. Power and Signature Sequence Allocation

When $K \leq N$, orthogonal sequences are generally employed. Our main interest lies in the case of $N < K < \frac{N}{e(\gamma)}$. Given a set of unit-energy signature sequences, the minimum user powers required to support all users with SIR $\gamma$ can be obtained following a derivation similar to the one in [6]:

$$\mathbf{p} = \left[ \frac{1}{e(\gamma)} \mathbf{I} - \mathbf{C} \right]^{-1} \tilde{\eta}.$$  

(5)

The structure of the optimal signature sequences, which consume the minimum total transmission power among all possible choice of unit-energy signature sequences, is currently unknown. Here, we propose two signature sequence allocation schemes to support users in the system.

<sup>2</sup>Inequality in (3) should be interpreted in the elementwise fashion.
A. Allocation Scheme I

One choice is the minimum-TSC sequences, i.e., WBE sequences. The WBE sequences are characterized by $SS^H = \frac{K}{N} I$, where $S = [s_1, s_2, \cdots, s_K]$ is an $N \times K$ matrix formed by grouping all $K$ spreading sequence vectors. The largest eigenvalue of $C$ is $\frac{K}{N}$ [8], which is smaller than $\frac{1}{\epsilon(\gamma)}$ by assumption. Hence, the target SIR $\gamma$ can be supported for all users.

With the WBE sequences as signature sequences, the total minimum transmission power required can be obtained as follows. Summing up all constraints in (2) with equality and making use of the uniformly-good property of the WBE sequences [8] that the sum of each column of $C$ is $\frac{K}{N}$, we have

$$P_{WBE} = \sum_{k=1}^{K} p_k = e(\gamma) \left( \sum_{l=1}^{K} \sum_{k=1}^{K} p_l c_{kl} + \tilde{\eta}_k \right)$$

$$= e(\gamma) \sum_{l=1}^{K} p_l \sum_{k=1}^{K} c_{kl} + \sum_{k=1}^{K} e(\gamma) \tilde{\eta}_k$$

$$= e(\gamma) P_{WBE} \frac{K}{N} + e(\gamma) \sum_{k=1}^{K} \tilde{\eta}_k. \quad (6)$$

This implies

$$P_{WBE} = \frac{N e(\gamma) \sum_{k=1}^{K} \tilde{\eta}_k}{N - K e(\gamma)}. \quad (7)$$

We note that the WBE sequences can be constructed iteratively [9], [10].

B. Allocation Scheme II

For convenience, we incorporate the user powers into the expressions of the sequences, and denote $a_k = \sqrt{p_k s_k}$ as the spreading sequences of the $k$th user to distinguish it from the unit-energy signature sequence $s_k$. Without loss of generality, define $\tilde{\eta}_0 = \infty$, and assume a descending order of the effective noise densities, i.e., $\tilde{\eta}_0 > \tilde{\eta}_1 \geq \tilde{\eta}_2 \geq \cdots \geq \tilde{\eta}_K$. There exists [4] a unique number $0 \leq k^* < N(1+\gamma) - K\gamma$ such that

$$\tilde{\eta}_{k^*+1} \leq \frac{\sum_{l=k^*+1}^{K} \tilde{\eta}_l}{N(1+\gamma) - K\gamma - k^*} < \tilde{\eta}_{k^*}. \quad (8)$$

We denote the first $k^*$ users, i.e., Users 1, 2, $\cdots$, $k^*$, as overfaded users, and Users $k^* + 1, k^* + 2, \cdots, K$ as nonoverfaded users. When $k^* = 0$, it means that there is no overfaded user. Now we introduce a power and sequence allocation scheme based on this classification. For convenience
we form an $N \times K$ matrix $A = [a_1, a_2, \ldots, a_K]$ by grouping all $K$ spreading sequence vectors. A feasible power and sequence allocation based on this classification is characterized by the following eigenvalue distribution of $A^H A$:

$$\lambda = (\tilde{p}_1, \cdots, \tilde{p}_{K^*}, \frac{\sum_{l=k^*+1}^{K} \tilde{p}_l}{N-k^*}, \cdots, \frac{\sum_{l=k^*+1}^{K} \tilde{p}_l}{N-k^*}, 0, \cdots, 0),$$

(9)

where

$$\tilde{p}_k = \begin{cases} \gamma \tilde{\eta}_k, & \text{for } k = 1, \cdots, k^*, \\ e(\gamma) \left( \frac{\gamma \sum_{l=k^*+1}^{K} \tilde{\eta}_l}{N(1+\gamma)-K\gamma-k} + \tilde{\eta}_k \right), & \text{for } k = k^*+1, \cdots, K, \end{cases}$$

(10)

are the user powers. Moreover,

$$A^H A = \begin{bmatrix} \sqrt{\tilde{p}_1} & & & \\ & \ddots & & \\ & & \sqrt{\tilde{p}_{k^*}} & \\ & & & 0 \\ & & & \bar{A}^H \bar{A} \end{bmatrix},$$

(11)

where $\bar{A}$ is an $N-k^* \times N-k^*$ matrix satisfying $\bar{A} \bar{A}^H = \frac{\gamma \sum_{l=k^*+1}^{K} \tilde{\eta}_l}{N(1+\gamma)-K\gamma-k^*}$ I. The resulting set of sequences are called the GWBE sequences. The total transmission power is

$$P_{GWBE} = \sum_{k=1}^{K} \tilde{p}_k = \gamma \sum_{l=1}^{k^*} \tilde{\eta}_l + \frac{(N-k^*) \gamma \sum_{l=k^*+1}^{K} \tilde{\eta}_l}{N(1+\gamma)-K\gamma-k^*}.$$  

(12)

The GWBE sequences characterized by (9)–(11) minimize the ETSC [4], which is defined as

$$ETSC = \sum_{k=1}^{K} \sum_{l=1}^{K} |\bar{a}_k^H s_l|^2 = \sum_{k=1}^{K} \sum_{l=1}^{K} |a_k^H a_l|^2.$$  

(13)

It can be shown [4] that the target SIR $\gamma$ is achieved for all users with this power and sequence allocation scheme. Under this allocation scheme, the overfaded users are allocated orthogonal “channels” and the nonoverfaded users share the remaining “channels”. When $K \leq N$ all users are overfaded according to (8) and are allocated orthogonal “channels”. This is consistent with the general orthogonal sequence allocation scheme for $K \leq N$. Like the WBE sequences, given the effective noise densities of all users, the GWBE sequences, i.e., $A$, can be constructed iteratively [4].
IV. PERFORMANCE ANALYSIS

High power efficiency is one of the main design concerns. In particular, the total transmission power should be minimized to reduce co-channel interference to other cells. In this section, we try to gauge the performance of the two power and sequence allocation schemes described previously in terms of the total transmission power required by each of the schemes.

A. Comparison of the Two Allocation Schemes

A simple algebra derivation shows that

\[
\frac{N\gamma \sum_{k=1}^{K} \tilde{\eta}_k}{N(1+\gamma) - K\gamma} \geq \frac{(N - k^*)\gamma \sum_{k=k^*+1}^{K} \tilde{\eta}_k}{(N(1+\gamma) - K\gamma) - k^*},
\]

with equality when \( k^* = 0 \). The inequality in (14) shows that the minimum-ETSC scheme is more power efficient than the minimum-TSC scheme.

B. Lagrangian Multiplier Search

We consider a Lagrangian multiplier method similar to the one in [11] to seek the minimum total transmission power numerically. The Lagrangian function \( L \) is formed as follows:

\[
L = \sum_{i=1}^{K} a_i^H a_i + \lambda \sum_{i=1}^{K} \left( \gamma - \frac{(a_i^H a_i)^2}{\sum_{m \neq i} |a_i^H a_m|^2 + \tilde{\eta}_k a_k^H a_k} \right)^2,
\]

where \( \lambda \) is the Lagrange multiplier. The first term on the left hand side of (15) represents the total transmission power of all users, while the second term enforces the SIR constraint in (1). Given the spreading gain \( N \), number of users \( K \), target SIR \( \gamma \), effective noise densities \( \tilde{\eta}_k \), for \( l = 1, 2, \cdots, K \), and Lagrange multiplier \( \lambda \), the Lagrangian function \( L \) is a function of components of \( a_l \) for \( 1 \leq l \leq K \). We consider a gradient search approach to seek a stationary point of the Lagrange function. The derivative of \( L \) with respect to \( a_k \) is given by

\[
\frac{\partial L}{\partial a_k} = 2a_k + 4\lambda \left[ \gamma - \frac{(a_k^H a_k)^2}{\sum_{m \neq k} |a_k^H a_m|^2 + \tilde{\eta}_k a_k^H a_k} \right] \cdot \\
+ 4\lambda \sum_{l \neq k} \frac{(a_l^H a_l)^2}{\sum_{m \neq l} |a_l^H a_m|^2 + \tilde{\eta}_l a_l^H a_l} \left( \sum_{m \neq l} (a_k^H a_m)a_m + \tilde{\eta}_k a_k^H a_k \right) \\
- \frac{2(a_k^H a_k)^2}{\sum_{m \neq k} |a_k^H a_m|^2 + \tilde{\eta}_k a_k^H a_k} \cdot \left( \sum_{m \neq k} |a_k^H a_m|^2 + \tilde{\eta}_k a_k^H a_k \right)^2.
\]
At each iteration of gradient search, $\mathbf{a}_k$ is updated using a gradient descent algorithm:

$$
\mathbf{a}_k \leftarrow \mathbf{a}_k - \mu \frac{\partial L}{\partial \mathbf{a}_k},
$$

for $k = 1, 2, \ldots, K$ with $\mu$ as the updating step size. The gradient descent algorithm may converge to local minima. Only the local minima which meet the target SIR of users are treated as valid searching results. After convergence, the total transmission power can be calculated as $P_{\text{Log}} = \sum_{k=1}^{K} \mathbf{a}_k^H \mathbf{a}_k$. The computation complexity of the algorithm per each search step is $O(NK^2)$. Although the complexity of this algorithm grows linearly in $N$ and quadratically in $K$ for each search step, the number of local minima could increase very significantly as $N$ and $K$ increase. This renders the Lagrangian search algorithm impractical for real-time applications.

We consider a sample system with spreading gain $N = 7$ and SIR target $\gamma = 4$dB. The user capacity of this sample system is 9 users according to (4). We conduct two simulation examples with $K = 8$ and $K = 9$ to check the power efficiency of the two allocation schemes. The inverse of effective noise densities $\frac{1}{\mathbf{l}_k}$, for $1 \leq k \leq K$, are generated independent exponential distributions with $E[\frac{1}{\mathbf{l}_k}] = 10$dB. For each example, we conduct 100 independent realizations and compare the best Lagrangian multiplier searching results $P_{\text{Log}}$ to the total transmission power of the minimum-TSC and minimum-ETSC allocation scheme. For ease of comparison, we normalize $P_{\text{GWBE}}$ and $P_{\text{WBE}}$ using $P_{\text{Log}}$. Simulation results are shown in Fig. 1. We can observe that the minimum-ETSC scheme is more power-efficient than the minimum-TSC scheme as indicated in (14). The power obtained using the Lagrangian searching method is very close to $P_{\text{GWBE}}$ for all the realizations conducted.

V. Conclusions

Two power and sequence allocation schemes are introduced to support all users with a uniform target SIR in a CDMA forward link. The minimum-ETSC scheme provides better power efficiency than the minimum-TSC scheme. In fact, from extensive simulations, it appears that the minimum-ETSC scheme is optimal in terms of requiring the minimum total transmission power. For a small system with $N = 2$ and $K = 3$, we conduct an exhaustive search over all the possible sequence structures to seek the one that gives the minimum total transmission power. The optimal sequence set obtained from the search has the same total squared correlation matrix $\mathbf{C}$ as that of the GWBE sequences for every channel realization that we have generated. How-
ever, analytical verification of the optimality of the minimum-ETSC allocation scheme remains open at this moment. We also note that one major difference between the proposed sequences and traditional spreading sequences is that the sequence elements are not restricted to a discrete alphabet.
REFERENCES


Fig. 1. Performance evaluation of WBE and GWBE sequences.