EEL6535: Digital Communications

Problem Set 2 (Spring 2004)

(due Class 11)

1. Proakis P5.2, 5.4, 5.8, and 5.10.

2. Binary data is to be transmitted over an AWGN channel with noise spectral density $N_0/2$ using the rectangular pulses $s_0(t) = Ap_T(t)$ and $s_1(t) = -Ap_T(t)$, where $p_T(t) = 1$ for $0 \leq t < T$ and $p_T(t) = 0$ otherwise. The receiver forms an output Z and decides "s_0 was sent" if $Z > 0$ and "s_1 was sent" if $Z \leq 0$.

   (a) Suppose that the output Z is given by

   $$Z = \int_0^T g(t)Y(t)dt,$$

   where $Y(t)$ is the transmitted signal plus noise and $g(t)$ is the triangular waveform

   $$g(t) = \begin{cases} t, & 0 \leq t < T/2, \\ T - t, & T/2 \leq t < T, \\ 0, & \text{otherwise.} \end{cases}$$

   Find the error probabilities $P_{b,0}$ and $P_{b,1}$.

   (b) Suppose the receiver produces the output $Z = Y(T/2)$; that is, it merely samples the received signal at time $T/2$. Find the error probabilities $P_{b,0}$ and $P_{b,1}$.

3. Consider the binary signal set $s_0(t) = Ap_T(t)$ and $s_1(t) = (-4At/T)p_T(t)$. Assume they are to be used to transmit binary data over an AWGN channel with spectral density $N_0/2$.

   (a) Find the matched filter for this signal set.

   (b) Assume the matched filter in (a) is employed. Find the optimum minimax threshold and the resulting optimum minimax error probability.

4. A binary baseband data transmission system uses the signal set consisting of $s_0(t) = s(t)$ and $s_1(t) = -s(t)$, where

   $$s(t) = \begin{cases} 2At/T, & 0 \leq t < T/2, \\ A(2t - T)/T, & T/2 \leq t < T, \\ 0, & \text{otherwise.} \end{cases}$$

   The channel is an AWGN channel with noise spectral density $N_0/2$. The minimax criterion is to be used.

   (a) What is the minimum error probability for this system? Give your answer in terms of A, T, N_0, and the function Q.

(b) Give the impulse response of the filter which achieves the minimum error probability. Simplify your answer as much as possible.

(c) Give the optimum sampling time and the optimum (minimax) threshold for the receiver which uses the filter of part (b).

(d) Find the variance σ^2 of the output process when the input to the filter of part (b) is a white noise process with spetral density $N_0/2$.

(e) Suppose that the filter in the receiver is not the filter of part (b), but instead a filter with impulse response $h(t) = p_T(t)$. Give an expression for the output $\hat{s}(t)$ of this filter when the input is $s(t)$.

(f) Repeat part (c) for the receiver which uses the filter of part (e).

(g) Find the bit error probability for the receiver which uses the filter of part (e) and the sampling time and threshold of part (f).