Homework 1
Due Wednesday, September 5, 2007

August 27, 2007

The following homework set has two types of problems. Those labeled “SS” are self-study problems that do not need to be turned in. However, there are problems that I suggest you complete to ensure that you understand all the material in the class. The problems not labeled SS should be turned for a grade. In general, the SS problems are easier than the graded problems, so they can be used to practice and build expertise before doing the graded problems.

1. Copy your student ID (preferred) or other photograph of yourself. Please make sure the picture is clear. Write on that sheet your full name, and the name you would like to be called. This page will not be returned, so please do not write anything else on that page.

On separate sheets of paper, do:

Sample Spaces, Events, and Sets

SS-1. A box contains three marbles, one red, one green, one blue.

(a) Consider an experiment that consists of taking 1 marble from the box, then replacing it in the box and drawing a second marble from the box. Describe the sample space.

(b) Repeat when the second marble is drawn without replacing the first marble.

SS-2. A die is rolled continually and the number on the top face is noted until a 6 appears, at which point the experiment stops.

(a) What is the sample space for this experiment?

(b) Let \( E_n \) denote the event that \( n \) rolls are necessary to complete the experiment. What points in the sample space are contained in \( E_n \)?

(c) What is 
\[
(\bigcup_{n=1}^{\infty} E_n)^c
\]  
(1)
SS-3. Two dice are thrown. Let $E$ be the event that the sum of the dice is odd; let $F$ be the event that at least one of the dice lands on 1; and let $G$ be the event that the sum is 5. Describe the events:

(a) $E \cap F$
(b) $E \cup F$
(c) $F \cap G$
(d) $E \cap \bar{F}$
(e) $E \cap F \cap G$

2. A hospital administrator codes incoming patients suffering from gunshot wounds according to whether they have insurance (coding 1 if they do and 0 if they do not) and according to their condition, which is rated as either good (g), fair (f), or serious (s). Consider an experiment that consists of coding such a patient.

(a) Give the sample space for the experiment.
(b) Let $A$ be the event that the patient is in serious condition. Specify the outcomes in $A$.
(c) Let $B$ be the event that the patient is uninsured. Specify the outcomes in $B$.
(d) Give all the outcomes in the event $B \cup A$.

3. For any sequence of events $E_1, E_2, \ldots$, define a new sequence $F_1, F_2, \ldots$ of mutually exclusive events such that for all $n \geq 1$,

$$
\bigcup_{i=1}^{n} F_i = \bigcup_{i=1}^{n} E_i.
$$

4.

Let $E$, $F$, and $G$ be three events. Find expressions for the events so that of $E$, $F$, and $G$:

(a) only $E$ occurs;
(b) both $E$ and $G$ but not $F$ occur;
(c) at least one of the events occurs;
(d) at least two of the events occur;
(e) all three occur;
(f) none of the events occurs;
(g) at most one of them occurs;
(h) at most two of them occur;
(i) exactly two of them occur;
(j) at most three of them occur.
SS-4. Suppose that $A$ and $B$ are mutually exclusive events for which $P(A) = 0.3$ and $P(B) = 0.5$. What is the probability that

(a) either $A$ or $B$ occurs;
(b) $A$ occurs but $B$ does not;
(c) both $A$ and $B$ occur?

5. A retail establishment accepts either the American Express or the VISA credit card. A total of 24 percent of its customers carry an American Express card, 61 percent carry a VISA card, and 11 percent carry both. What percentage of its customers carry a card that the establishment will accept?

SS-5. A total of 28 percent of American males smoke cigarettes, 7 percent smoke cigars, and 5 percent smoke both cigars and cigarettes.

(a) What percentage of males smoke neither cigars nor cigarettes?
(b) What percentage smoke cigars but not cigarettes?

6. An elementary school is offering 3 language classes: one in Spanish, one in French, and one in German. These classes are open to any of the 100 students in the school. There are 28 students in the Spanish class, 26 in the French class, and 16 in the German class. There are 12 students that are in both Spanish and French, 4 that are in both Spanish and German, and 6 that are in both French and German. In addition, there are 2 students taking all 3 classes.

(a) If a student is chosen randomly, what is the probability that he or she is not in any of these classes?
(b) If a student is chosen randomly, what is the probability that he or she is taking exactly one language class?
(c) If two students are chosen randomly, what is the probability that at least one is taking a language class?