Special Problems for HW 3
September 23, 2002

These are some former exam problems to give you an idea of my test problems.

SP2. In the 17th century, a French aristocrat named the Chevalier de Méré developed several games of chance based on dice. In the first game he developed, he would bet even money that at least one 6 would show up on four rolls of a 6-sided die.

His reasoning was that $P(6) = \frac{1}{6}$, so $P(\text{one 6 in four rolls}) = 4 \times \frac{1}{6} = \frac{4}{6}$.

(a) Was the Chevalier’s reasoning correct? Explain your answer.

(b) If not, what is the probability of exactly one 6 in four rolls of a die?

(c) What is the probability of at least one 6 in four rolls of a die?

Even money means that for each amount bet on getting at least one 6, if a six shows up, the bettor receives his original bet back plus winnings equal to the original bet. In an even-money bet, the gambler wins in the long run if the event he wagers (bets) on occurs with probability greater than 0.5.

(d) Did the Chevalier win money playing this game?

When no one would take the Chevalier’s bet anymore, he developed a second game. He bet that double-6 would show up at least once in 24 rolls of two dice. His reasoning was that $P(6, 6) = \frac{1}{36}$, so $P(6, 6 \text{ in 24 rolls}) = 24 \times \frac{1}{36} = \frac{2}{3}$.

(e) Was the Chevalier’s reasoning correct?

(f) If not, what is the probability of at least one occurrence of double-6 in 24 rolls of two dice?

(g) For the even-money wager, did the Chevalier win money playing the new game?
You are sent as part of a UN inspection team to investigate whether Saddam Hussein has weapons-grade plutonium (WGP) in a reactor. Let $W$ denote the event that WGP is present.

You take measurements with a Geiger counter and decide that WGP is detected if some threshold is exceeded. Suppose that the threshold is set so that the probability of correct detection, $P(D|W)$, is 0.9, and the probability of false detection, $P(D|\overline{W})$, is 0.25.

Suppose that the probability that WGP is present is 0.3.

It may help to draw a diagram showing the transition probabilities.

(a) What is the probability that WGP is detected? Not detected?

(b) If you detect WGP, what is the probability that it is actually present?

(c) If you detect WGP, what is the probability that it is not present? (The conditional probability of false alarm).

(d) If you do not detect WGP, what is the probability that it was actually there? (The conditional probability of a miss).

(e) What is the overall probability that you make the wrong conclusion based on your measurements?