1. Define \( \triangle(t) \) (read “triangle of \( t \)”’) by

\[
\triangle(t) = \begin{cases} 
    t + 1, & -1 \leq t < 0 \\
    1 - t, & 0 \leq t \leq 1 \\
    0, & \text{otherwise.}
\end{cases}
\]

Suppose \( m(t) \) is the waveform given by

\[
m(t) = \sum_{k=-\infty}^{\infty} \left[ \triangle \left( \frac{t - 1}{2} - 2k \right) - \frac{1}{2} \right]
\]

and \( m(t) \) is used to amplitude modulate a carrier such that the output signal is given by

\[
s_{AM}(t) = 10 \left[ 1 + m(t) \right] \cos(200\pi t).
\]

(a) Sketch \( m(t) \) from \( t = 0s \) to \( t = 10s \).

(b) Sketch the envelope of \( s_{AM}(t) \). Label relevant amplitudes and times.

(c) Sketch roughly \( s_{AM}(t) \). You may overlay this sketch with that in part (b) if you label each part clearly. Do not try to show every cycle of the carrier waveform – just give the general idea.

(d) Find the modulation index and percent modulation for \( s_{AM}(t) \).

(e) Find the total average power for \( s_{AM}(t) \).

(f) Find the carrier power.

(g) Find the modulation efficiency.

(h) Suppose you did not know \( m(t) \) but knew the percent modulation was the value you found in part (d). Calculate the efficiency under the assumption that \( m(t) \) is a sinusoid. Compare your answer to your answer for part (f).

2. Suppose a sinusoid of amplitude 4 V and frequency 1 kHz is used to modulate a carrier of frequency 100 kHz and amplitude 40 V. The FM modulator has a modulator gain (i.e., frequency deviation constant) of 50 Hz/V.

(a) What is the peak frequency deviation?

(b) What is the modulation index?

(c) Use Carson’s rule to find upper and lower bounds for the 98% bandwidth, \( B_T \).

(d) Assume that \( B_T \) is given by the upper bound. Find the frequency ranges for the upper and lower sidebands.

(e) Estimate the total transmitted power.