On the Asymptotic Performance of Threshold-based Acquisition Systems in Multipath Fading Channels

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Abstract—In this paper, the asymptotic performance of timing acquisition systems having fixed dwell time in multipath fading channels is investigated. The detrimental effect of the multipath channel fading on the acquisition performance is isolated by considering the asymptotic performance as the average signal-to-noise ratio (SNR) increases without bound. It is found that for any threshold such that the average probability of false alarm is less than a given tolerance, the channel fading results in a lower bound on the asymptotic average probability of miss which is non-trivial for a variety of fading scenarios. A threshold-based direct-sequence spread-spectrum signal acquisition system is considered and it is found that the detrimental effect of channel fading on asymptotic acquisition performance, albeit non-trivial, is not very significant. The asymptotic acquisition performance of two threshold-based acquisition schemes for ultra-wideband signals with time-hopping spreading are also evaluated and compared. For both the schemes, the detrimental effect of the channel fading on the asymptotic acquisition performance turns out to be significant.

Index Terms—Acquisition, asymptotic performance, CDMA, fading channels, ultra-wideband.

I. INTRODUCTION

In any communication system, the acquisition of the timing of the received signal is the first operation which needs to be performed before any further processing of the received signal can be done. Although the ideal method of performing acquisition is a likelihood-ratio test or a uniformly most powerful test [1], most practical acquisition systems are implemented as a simple energy detector. A typical timing acquisition system consists of a stage which generates a template signal with a hypothesized value for the symbol timing and correlates it with the received signal to obtain a decision statistic [2]. This decision statistic is then compared to a threshold to determine if the hypothesized value for the symbol timing is correct or not. The threshold is chosen such that, with high probability, it is larger than the value of the decision statistic when the hypothesized symbol timing is correct. Hence, the threshold represents the case of packetized mobile communication systems. This is a scenario where good acquisition performance is crucial, since the timing needs to be repeatedly estimated for every packet as it may change due to node mobility. And since throughput considerations limit the length of the preamble which can be prepended to a particular packet, there might be a limit to the accuracy with which the timing can be estimated. Thus it is of interest to get an estimate of the best possible acquisition performance which can be achieved by using a finite-length preamble.

In the absence of channel fading, it is a well-known result that the probabilities of occurrence of false alarms and misses, which are due to the noise alone, can be made arbitrarily small by operating at a higher SNR, which is typically done by increasing the dwell time of the correlator [2]. As the SNR increases, even a sub-optimally chosen threshold, located between the means of the distributions of the decision statistic when the hypothesized symbol timing is correct and incorrect, forces the probabilities of false alarm and miss to become arbitrarily small. It is, however, reasonable to expect that the presence of channel fading can cause errors to occur, irrespective of how high the average SNR is. This is due to the fact that a high average SNR only guarantees that the detrimental effect of the noise is negligible and the channel fading can still induce errors in the acquisition process.

In this paper, we investigate the asymptotic error performance of threshold-based timing acquisition systems having fixed dwell time in multipath fading channels. We restrict our attention to acquisition systems with fixed dwell time because it represents the case of packetized mobile communication systems. This is a scenario where good acquisition performance is crucial, since the timing needs to be repeatedly estimated for every packet as it may change due to node mobility. And since throughput considerations limit the length of the preamble which can be prepended to a particular packet, there might be a limit to the accuracy with which the timing can be estimated. Thus it is of interest to get an estimate of the best possible acquisition performance which can be achieved by using a finite-length preamble.

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In this paper, we isolate the detrimental effect of the multipath channel fading on the acquisition performance of a finite dwell time threshold-based acquisition system, by considering the asymptotic performance as the average SNR increases without bound. We show that no matter how large the average SNR is or how we choose the threshold, there exist fading scenarios with a non-zero and sometimes restrictive average probability of occurrence of false alarms and misses.

We describe the system model in Section II which is general enough to encompass most threshold-based timing acquisition systems. In Section III, we state and prove the main result of the paper which basically says that if there is a threshold which restricts the average probability of false alarm to be smaller than a fixed tolerance, then no matter how large the average SNR is, there is a possibly non-trivial lower bound on the asymptotic average probability of miss. In Section IV,
we apply the result to a direct-sequence spread-spectrum acquisition system and evaluate the asymptotic performance for both Rician- and Rayleigh-fading channels. In Section V, we evaluate and compare the asymptotic acquisition performance of two acquisition schemes for ultra-wideband signals with time-hopping spreading. Section VI has some discussion and conclusions.

II. SYSTEM MODEL

Let $s(t)$ be the transmitted signal and $h(t)$ be the channel response which is assumed to be random but fixed during the acquisition process. Then the received signal is given by $r(t) = s(t - \tau) + n(t)$ where $s(t) = s(t) * h(t)$ ($*$ denotes convolution), $\tau$ is the true symbol timing and $n(t)$ is a zero-mean wide-sense stationary (WSS) additive noise process. Let $\hat{\tau}$ be the hypothesized symbol timing. Then the decision statistic generated by the acquisition system is given by $R(\Delta \tau; h) = g(r(t), \hat{\tau})$ where $g$ is some bivariate functional, $\Delta \tau = \hat{\tau} - \tau$ and assuming that the channel fading effects can be characterized by a finite-dimensional vector $h$. Let $F_N(\cdot; \Delta \tau|h)$ be the conditional cumulative distribution function (CDF) of $R(\Delta \tau; h)$ conditioned on $h$.

In a multipath channel, the receiver need not lock to the line-of-sight (LOS) path to perform successful demodulation. The energy in the non-LOS multipaths may be enough to enable a receiver which locks to them to perform adequately. Depending on the performance criteria chosen, there will be a set of hypothesized symbol timings $\hat{\tau}$ called the hit set (which we will denote by $S_h$) where a receiver lock can be considered successful acquisition. Since the goal of the acquisition process is to achieve coarse synchronization, the true symbol timing $\tau$ can be assumed to belong to a finite set $S_\tau$ of symbol times which is an adequately quantized version of the timing ambiguity region. The hypothesized symbol timings are chosen from this finite set and hence the hit set is also finite. Note that it is the distance $\Delta \tau$ of a hypothesized symbol timing $\hat{\tau}$ from the true symbol timing $\tau$ which determines if $\hat{\tau}$ belongs to $S_h$ or not. In this sense, the actual value of the true symbol timing is irrelevant. For a particular value of $\tau$, the acquisition process can be formulated as a composite binary hypothesis testing problem with the following hypotheses:

$$
H_0 : \hat{\tau} \notin S_h \\
H_1 : \hat{\tau} \in S_h.
$$

The probabilities of false alarm and detection conditioned on the particular channel realization $h$ and given the decision threshold $\gamma$ are given, respectively, by

$$
P_{FA}(\gamma; \Delta \tau|h) = \Pr[R(\Delta \tau; h) > \gamma| h, \hat{\tau} \notin S_h] = 1 - F_N(\gamma; \Delta \tau|h) = F'_N(\gamma; \Delta \tau|h), \hat{\tau} \notin S_h, \quad (1)$$

$$
P_{D}(\gamma; \Delta \tau|h) = \Pr[R(\Delta \tau; h) > \gamma| h, \hat{\tau} \in S_h] = 1 - F_N(\gamma; \Delta \tau|h) = F'_N(\gamma; \Delta \tau|h), \hat{\tau} \in S_h, \quad (2)$$

where $F'_N(\cdot; \Delta \tau|h)$ is the complementary conditional CDF of $R(\Delta \tau; h)$ conditioned on $h$. Then the probabilities of false alarm and detection averaged over the channel realizations are given by

$$
P_{FA}(\gamma; \Delta \tau) = E_h[P_{FA}(\gamma; \Delta \tau|h)] = E_h[F'_N(\gamma; \Delta \tau|h)], \hat{\tau} \notin S_h, \quad (4)$$

$$
P_{D}(\gamma; \Delta \tau) = E_h[P_{D}(\gamma; \Delta \tau|h)] = E_h[F'_N(\gamma; \Delta \tau|h)], \hat{\tau} \in S_h. \quad (5)$$

The average probability of miss is then given by

$$
P_M(\gamma; \Delta \tau) = 1 - P_D(\gamma; \Delta \tau), \hat{\tau} \notin S_h. \quad (6)$$

Henceforth, whenever we write $P_{FA}(\gamma; \Delta \tau)$ or $P_{FA}(\gamma; \Delta \tau|h)$ it is implicit that $\hat{\tau} \notin S_h$. Similarly, $P_D(\gamma; \Delta \tau)$, $P_D(\gamma; \Delta \tau|h)$, $P_M(\gamma; \Delta \tau)$ and $P_M(\gamma; \Delta \tau|h)$ all imply that $\hat{\tau} \in S_h$.

III. ASYMPTOTIC PERFORMANCE OF THRESHOLD-BASED ACQUISITION SYSTEMS

Let $\sigma^2$ be the power (variance) of the noise process $n(t)$. Let $\mathcal{H}$ be the set of all possible channel parameter vectors $h$. Note that $P_{FA}(\gamma; \Delta \tau)$ and $P_M(\gamma; \Delta \tau)$ defined in the previous section are functions of $\sigma$. To avoid cumbersome notation, we write $\lim_{\sigma \to 0^+} P_{FA}(\gamma; \Delta \tau)$ to mean $\lim_{\sigma \to 0^+} P_{FA}(\gamma, \sigma; \Delta \tau)$. Furthermore, for a positive sequence $\{\sigma_n\}$ with limit $0$, we write $\lim \sup_{\sigma_n \to 0^+} P_M(\gamma; \Delta \tau)$ to mean $\lim \sup_{\sigma_n \to 0^+} P_M(\gamma, \sigma_n; \Delta \tau)$. The following theorem is the main result of this paper.

**Theorem 1:** Consider a threshold-based acquisition system with decision statistic $R(\Delta \tau; h)$ with the property that for every threshold $\gamma$ and $\epsilon > 0$, there is an $\eta(\gamma, \epsilon) > 0$ such that when $\sigma^2 < \eta(\gamma, \epsilon)$ there exist subsets $\mathcal{A}_\epsilon(\gamma; \Delta \tau), \mathcal{B}_\epsilon(\gamma; \Delta \tau)$ of $\mathcal{H}$ for every $\hat{\tau} \in S_h$, such that

(i) $\Pr(\mathcal{A}_\epsilon(\gamma; \Delta \tau) \cup \mathcal{B}_\epsilon(\gamma; \Delta \tau)) > 1 - \epsilon$.

(ii) For all $h \in \mathcal{A}_\epsilon(\gamma; \Delta \tau)$, $F_N(\gamma; \Delta \tau|h) > 1 - \epsilon$.

(iii) For all $h \in \mathcal{B}_\epsilon(\gamma; \Delta \tau)$, $F_N(\gamma; \Delta \tau|h) \leq \epsilon$.

For some $\delta > 0$, if there exists an $\eta_\delta(\delta) > 0$ such that $P_{FA}(\gamma; \Delta \tau) < \delta$ for all $\sigma^2 < \eta_\delta(\delta)$ and for all $\hat{\tau} \notin S_h$, then $\lim_{\sigma \to 0^+} P_M(\gamma; \Delta \tau) \geq \lim_{\sigma \to 0^+} \Pr(\mathcal{A}_\epsilon(\gamma; \Delta \tau))$ where $\gamma_\delta(\delta) = \inf\{\gamma : \lim_{\sigma \to 0^+} \Pr(\mathcal{B}_\epsilon(\gamma; \Delta \tau)) \leq \delta, \text{ for all } \hat{\tau} \notin S_h\}$. Furthermore, given $\xi > 0$ there exists a $\kappa(\delta, \xi) > 0$ such that $P_M(\gamma; \Delta \tau) \geq \lim_{\sigma \to 0^+} \Pr(\mathcal{A}_\epsilon(\gamma_\delta(\delta); \Delta \tau)) - \xi$ for all $\sigma^2 < \kappa(\delta, \xi)$.

**Proof:** See Appendix A.

We present some discussion regarding the conditions and statement of the above theorem. From (2) and (3), it is clear that the set $\mathcal{A}_\epsilon(\gamma; \Delta \tau)$ corresponds to a subset of $\mathcal{H}$ where $P_{FA}(\gamma; \Delta \tau|h)$ or $P_D(\gamma; \Delta \tau|h)$ (depending on whether $\hat{\tau} \notin S_h$ or $\hat{\tau} \in S_h$) do not exceed $\epsilon$. Similarly, the set $\mathcal{B}_\epsilon(\gamma; \Delta \tau)$ corresponds to a subset of $\mathcal{H}$ where $P_{FA}(\gamma; \Delta \tau|h)$ or $P_D(\gamma; \Delta \tau|h)$ exceed $1 - \epsilon$. So the conditions of Theorem 1 require the decision statistic to be such that when the noise variance is small enough (or equivalently at high enough SNRs), the conditional probabilities of false alarm and detection are (with probability close to one) either close to zero or close to one. Furthermore, using conditions (i)-(iii) of the theorem and (4)-(5), it is easy to see that at high SNRs $P_{FA}(\gamma; \Delta \tau) \approx \Pr(\mathcal{B}(\gamma; \Delta \tau))$ and $P_M(\gamma; \Delta \tau) \approx \Pr(\mathcal{A}(\gamma; \Delta \tau))$. Any threshold $\gamma$ which restricts
\( P_{FA}(\gamma; \Delta \tau) \) to be less than some \( \delta \) will be larger than the smallest threshold \( \gamma_m(\delta) \) which restricts \( \Pr(B_c(\gamma; \Delta \tau)) \) to not exceed \( \delta \). So the theorem states that this lower bound on the threshold translates to a lower bound on \( P_M(\gamma; \Delta \tau) \) which may be non-trivial even in the asymptotic scenario. If the threshold \( \gamma \) is chosen carefully, then we have \( P_M(\gamma; \Delta \tau) \geq \lim_{\Delta \tau \to 0^+} P_M(\gamma; \Delta \tau) \), but this is not true in general for all \( \gamma \). So the lower bound on the asymptotic average probability of miss may not always be a lower bound on the average probability of miss at finite SNRs. Nevertheless, the last statement of the theorem states that the lower bound in the asymptotic case is a good approximation for the lower bound on the average probability of miss at large (finite) SNRs. Thus the tradeoff between the \( P_{FA}(\gamma; \Delta \tau) \) and \( P_M(\gamma; \Delta \tau) \) at large SNRs can be characterized by the tradeoff between \( \delta \) and \( \lim_{\Delta \tau \to 0^+} \Pr(A_c(\gamma_m(\delta); \Delta \tau)) \). The main advantage of the theorem is that this tradeoff can be calculated using sets defined according to the conditional probabilities of detection and false alarm, which are usually easier to obtain.

IV. ACQUISITION IN A DIRECT-SEQUENCE SPREAD-SPECTRUM SYSTEM

We consider the acquisition of the pseudo-noise (PN) spreading sequence in a direct sequence spread-spectrum (DS-SS) system and evaluate the bounds discussed in Theorem 1 for different channel models.

A. DS-SS Signal Acquisition System

The transmitted signal is given by

\[
s(t) = \sqrt{2P} \sum_{i=-\infty}^{\infty} c_i p(t - iT_c) \cos \omega_0 t, \tag{7}
\]

where \( P \) is the transmitter signal power, \( \omega_0 \) is the carrier frequency in rad/s, \( p(t) \) is the chip waveform normalized to have unit energy and \( c_i \) is the periodic PN sequence of period \( N \) taking values in \([-1,+1]\). We assume a stochastic tapped delay line model for the frequency-selective fading channel with tap spacing equal to a chip duration \( T_c \) [3] where the fading is assumed to be constant over the dwell time of the acquisition system. Then the received signal is given by

\[
r(t) = \sqrt{2P} \sum_{i=-\infty}^{\infty} \sum_{k=0}^{N_{tap}-1} h_k c_i p(t - (i - k)T_c) \times \cos(\omega_0 t + \theta_k) + n(t), \tag{8}
\]

where \( P_r \) is the received signal power, \( N_{tap} \) is the number of resolvable received paths, \( h_k \) and \( \theta_k \) are the attenuation and phase of the \( k \)th path and \( n(t) \) is an additive white Gaussian noise (AWGN) process with zero mean and power spectral density \( \sigma^2 \). The total power in all the resolvable paths is normalized to unity i.e. \( \sum_{k=0}^{N_{tap}-1} E[h_k^2] = 1 \). We assume that the receiver is chip-synchronized to the received signal and since the ambiguity region is equal to the period of the sequence \( NT_c \), the search space \( S_p \) consists of \( N \) code phases. A fixed dwell non-coherent detection scheme similar to the one described in [3] is used to generate the decision statistic. The non-coherent detection system consists of an in-phase and a quadrature branch each of which contains a chip-matched filter whose output is sampled at the chip rate and correlated with \( M \) chips of the locally generated PN sequence. The correlator outputs of the in-phase and quadrature branches are squared and summed to get the decision statistic. If the phase offset between the true code phase \( \tau \) and the hypothesized phase \( \hat{\tau} \) is \( \Delta \tau = nT_c \), the decision statistic is given by

\[
R(\Delta \tau; h) = V_c^2(n) + V_q^2(n), \tag{9}
\]

where \( h \) is a vector containing the channel attenuations and phases \{\( h_k, \theta_k \)\}_{k=0}^{N_{tap}-1} and \( V_c(n), V_q(n) \) are the outputs of the correlators in the in-phase and quadrature branches respectively. They are given by

\[
V_c(n) = \sqrt{P_r} \sum_{k=0}^{N_{tap}-1} r_c(n + k) h_k \cos \theta_k + n_c,
\]

\[
V_q(n) = \sqrt{P_r} \sum_{k=0}^{N_{tap}-1} r_c(n + k) h_k \sin \theta_k + n_s, \tag{10}
\]

where \( r_c(n) = \frac{1}{M} \sum_{i=0}^{M-1} c_i c_{i+n} \) is the partial autocorrelation function of the PN sequence and \( n_c, n_s \) are uncorrelated zero-mean Gaussian random variables with variance \( \sigma^2 \). We assume a random sequence model for the PN sequence where each sequence element \( c_i \) is i.i.d. and equally likely to be \(+1\) or \(-1\). We know that \( r_c(0) = 1 \). For \( n \neq 0 \), if \( M \) is large then \( r_c(n) \approx 0 \) and \( r_c^2(n) \approx \frac{1}{M} \).

If \( \tau \) is the true code phase, then we take the hit set \( S_h \) to be the set \{\( \tau, \tau + T_c, \ldots, \tau + (N_{tap}-1)T_c \)\} which are the phases containing a resolvable path corresponding to the true phase of the received signal [4],[5]. This is not a reasonable definition for the hit set at finite SNRs since some of the resolvable paths may be too weak to enable good demodulation performance. Hence a receiver lock to such a path may not be considered successful acquisition. However, \( S_{h'} \) defined as above contains any path where good demodulation performance can be achieved at a finite SNR. Thus it represents the largest possible hit set and consequently \( S_{h'} \) is the smallest possible non-hit set. This corresponds to the least restrictive choice of \( \gamma_m(\delta) \) in Theorem 1. For this choice of \( S_{h'} \), the lower bound on the asymptotic average probability of miss is the smallest and hence it results in the best possible asymptotic acquisition performance over all choices of \( S_{h'} \).

The decision statistic \( R(\Delta \tau; h) \) conditioned on the particular channel realization \( h \) has a non-central chi-square PDF with 2 degrees of freedom [6]. Then for a given threshold \( \gamma \), the conditional probabilities of false alarm and detection are given by

\[
P_{FA}(\gamma; \Delta \tau | h) = F_N(\gamma; \Delta \tau | h), \hat{\tau} \notin S_h,
\]

\[
= Q_{1}\left( \frac{\gamma}{\sigma}, \frac{\sqrt{\gamma}}{\sigma} \right), \hat{\tau} \notin S_h,
\]

\[
P_{D}(\gamma; \Delta \tau | h) = F_N(\gamma; \Delta \tau | h), \hat{\tau} \in S_h,
\]

\[
= Q_{1}\left( \frac{\gamma}{\sigma}, \frac{\sqrt{\gamma}}{\sigma} \right), \hat{\tau} \in S_h, \tag{11}
\]

where \( Q_{1}(\cdot, \cdot) \) is the generalized Marcum’s Q-function and for
\[ \Delta \tau = nT_c \text{ we have} \]
\[ s^2(\Delta \tau; \mathbf{h}) = \left( \sqrt{P_r} \sum_{k=0}^{N_{\text{tap}}-1} r_c(n+k)h_k \cos \theta_k \right)^2 + \left( \sqrt{P_r} \sum_{k=0}^{N_{\text{tap}}-1} r_c(n+k)h_k \sin \theta_k \right)^2 \]
\[ = P_r \sum_{k=0}^{N_{\text{tap}}-1} \left[ \frac{1}{M} + \left( 1 - \frac{1}{M} \right) \delta_D(n+k) \right] h_k^2, \]
\[ (12) \]

where \( \delta_D(\cdot) \) is the Kronecker delta function. The second equality in the above equation is due to the approximation of the partial autocorrelation function discussed earlier.

### B. Asymptotic Performance of DS-SS Signal Acquisition Systems

In order to be able to apply Theorem 1 to this case, we need to first verify that the required conditions hold. We assume that the channel parameter vector \( \mathbf{h} \) has an absolutely continuous distribution [7], which is true for the fading channel models usually considered for DS-SS systems. Hence \( s(\Delta \tau; \mathbf{h}) \) has an absolutely continuous distribution for every \( \hat{\tau} \in S_p \). Since \( S_p \) is finite, for every threshold \( \gamma \geq 0 \) and an \( \epsilon > 0 \), there exists a \( \kappa(\gamma, \epsilon) > 0 \) such that for all \( \hat{\tau} \in S_p \),
\[ \Pr(\{ \mathbf{h} : \sqrt{\gamma} - \kappa(\gamma, \epsilon) \leq s(\Delta \tau; \mathbf{h}) \leq \sqrt{\gamma} + \kappa(\gamma, \epsilon) \}) < \epsilon. \]
\[ (13) \]

In Appendix B, we show that \( A_c(\gamma; \Delta \tau) \) and \( B_c(\gamma; \Delta \tau) \) defined below satisfy the conditions of Theorem 1,
\[ A_c(\gamma; \Delta \tau) = \{ \mathbf{h} : s(\Delta \tau; \mathbf{h}) < \sqrt{\gamma} - \kappa(\gamma, \epsilon) \}. \]
\[ B_c(\gamma; \Delta \tau) = \{ \mathbf{h} : s(\Delta \tau; \mathbf{h}) > \sqrt{\gamma} + \kappa(\gamma, \epsilon) \}. \]
\[ (14) \]

Then for all \( \epsilon > 0 \),
\[ \Pr( A_c(\gamma; \Delta \tau) ) = \Pr( \{ \mathbf{h} : s(\Delta \tau; \mathbf{h}) \leq \sqrt{\gamma} \} ) - \Pr( \{ \mathbf{h} : s(\Delta \tau; \mathbf{h}) > \sqrt{\gamma} \} ) < \epsilon, \]
\[ \geq \Pr( \{ \mathbf{h} : s(\Delta \tau; \mathbf{h}) \leq \sqrt{\gamma} \} ) - \Pr( \{ \mathbf{h} : s(\Delta \tau; \mathbf{h}) > \sqrt{\gamma} + \kappa(\gamma, \epsilon) \} ) \]
\[ > \Pr( \{ \mathbf{h} : s(\Delta \tau; \mathbf{h}) \leq \sqrt{\gamma} \} ) - \epsilon, \]
\[ (15) \]

where the last inequality follows from (13). Since \( \Pr( A_c(\gamma; \Delta \tau) ) \) \( \leq \Pr( \{ \mathbf{h} : s(\Delta \tau; \mathbf{h}) \leq \sqrt{\gamma} \} ) \) for all \( \epsilon > 0 \), we have
\[ \lim_{\epsilon \to 0^+} \Pr( A_c(\gamma; \Delta \tau) ) = \Pr( \{ \mathbf{h} : s(\Delta \tau; \mathbf{h}) \leq \sqrt{\gamma} \} ) = \Pr( \{ \mathbf{h} : s^2(\Delta \tau; \mathbf{h}) \leq \gamma \} ). \]
\[ (16) \]

Similarly, we can show that
\[ \lim_{\epsilon \to 0^+} \Pr( B_c(\gamma; \Delta \tau) ) = \Pr( \{ \mathbf{h} : s(\Delta \tau; \mathbf{h}) \geq \sqrt{\gamma} \} ). \]
\[ (17) \]

Then by Theorem 1, for any \( \delta > 0 \) if there exists a threshold \( \gamma \) and an \( \eta_1(\delta) > 0 \) such that \( P_{\text{FA}}(\gamma; \Delta \tau) < \delta \) for all \( \sigma^2 < \eta_1(\delta) \) and for all \( \hat{\tau} \notin S_h \), then
\[ \lim_{\sigma \to 0^+} P_{\text{M}}(\gamma; \Delta \tau) \geq \Pr( \{ \mathbf{h} : s^2(\Delta \tau; \mathbf{h}) \leq \gamma_m(\delta) \} ). \]
\[ (18) \]

where \( \gamma_m(\delta) = \inf \{ \gamma : \Pr( \{ \mathbf{h} : s^2(\Delta \tau; \mathbf{h}) \geq \gamma \} ) \leq \delta, \text{ for all } \hat{\tau} \notin S_h \} \).

Note that the lower bound on the asymptotic average probability of miss in (18) results in the following upper bound on the asymptotic average probability of detection,
\[ \lim_{\sigma \to 0^+} P_D(\gamma; \Delta \tau) \leq \Pr( \{ \mathbf{h} : s^2(\Delta \tau; \mathbf{h}) \geq \gamma_m(\delta) \} ). \]
\[ (19) \]

where \( \hat{\tau} \in S_h \). By evaluating this upper bound as a function of \( \delta \), we obtain an asymptotic receiver operating characteristic (AROC) which characterizes the best achievable trade-off between the average probabilities of false alarm and detection. From the definition of \( \gamma_m(\delta) \) and the expression for the upper bound in (19), we observe that the AROC for a particular \( \hat{\tau} \in S_h \) depends on the separation between the corresponding distribution of \( s^2(\Delta \tau; \mathbf{h}) \) and the distributions of \( s^2(\Delta \tau; \mathbf{h}) \) for all \( \hat{\tau} \notin S_h \). For instance, if the distribution of \( s^2(\Delta \tau; \mathbf{h}) \) for some \( \hat{\tau} \in S_h \) is close to the distribution of \( s^2(\Delta \tau; \mathbf{h}) \) for any \( \hat{\tau} \notin S_h \), then the upper bound on the asymptotic average probability of detection for that \( \hat{\tau} \in S_h \) will be close to \( \delta \).

We evaluate the AROC for both Ricean- and Rayleigh-fading channels. From (12), we note that the AROC does not depend on the received power \( P_r \), since it is only a scaling factor and hence we can set \( P_r = 1 \). For the Ricean-fading channel model, we assume that the fading amplitudes are independently distributed and that there is a specular component only in the first path [8]. So the fading amplitude of the first resolvable path has a Rayleigh distribution while the amplitudes of the other resolvable paths are Rayleigh distributed. The diffuse power in the resolvable multipaths is assumed to follow an exponential decay with rate \( \mu \). If \( P_\alpha^2 \) is the specular-to-total diffuse power ratio, the average power in the multipath components is given by
\[ E[h_k^2] = \begin{cases} \frac{P_\alpha^2 + C}{1 + P_\alpha^2} & \text{if } k = 0, \\ \frac{C e^{-\mu}}{1 + P_\alpha^2} & \text{for } k = 1, 2, \ldots, N_{\text{tap}} - 1, \end{cases} \]
\[ (20) \]

where \( C = \frac{1 - e^{-\mu}}{1 - e^{-\mu} N_{\text{tap}}} \) is a constant chosen to ensure that \( \sum_{k=0}^{N_{\text{tap}}-1} E[h_k^2] = 1 \). In this case, \( s^2(\Delta \tau; \mathbf{h}) \) is a weighted sum of a non-central chi-square distributed random variable with 2 degrees of freedom having characteristic function
\[ \phi_0(\omega) = \frac{1}{1 - j\omega} \exp \left( \frac{j\omega P_\alpha^2}{1 + P_\alpha^2} \right), \]
\[ (21) \]

and \( N_{\text{tap}} - 1 \) independent and possibly non-identically distributed central chi-square random variables, each with 2 degrees of freedom and having characteristic functions
\[ \phi_k(\omega) = \frac{1}{1 - j\omega E[h_k^2]} \]
\[ (22) \]

for \( k = 1, 2, \ldots, N_{\text{tap}} - 1 \). Then the characteristic function of \( s^2(\Delta \tau; \mathbf{h}) \) is given by
\[ \phi_s(\omega; \Delta \tau) = \prod_{k=0}^{N_{\text{tap}}-1} \phi_k \left( \left[ \frac{1}{M} + \left( 1 - \frac{1}{M} \right) \delta_D(n+k) \right] \omega \right). \]
\[ (23) \]

\(^1\)The random variables are identically distributed if \( \mu = 0 \). In this case, \( C = \lim_{\mu \to 0} \frac{1 - e^{-\mu}}{1 - e^{-\mu} N_{\text{tap}}} = \frac{1}{N_{\text{tap}}} \).
By the Gil-Pelaez lemma [9], the CDF of \( s^2(\Delta \tau; h) \) is given by

\[
F_s(x) = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Im} \left\{ \frac{e^{i\pi \phi_s(-t; \Delta \tau)}}{t} \right\} dt = \frac{1}{2} + \frac{2}{\pi} \int_0^{\pi/2} \text{Im} \left\{ \frac{e^{i\pi \tan \theta \phi_s(-\tan \theta; \Delta \tau)}}{\sin 2\theta} \right\} d\theta,
\]

(24)

where the second equality is obtained by the change of variable \( t = \tan \theta \). The second integral has finite limits of integration and hence is more suitable for numerical evaluation.

The Rayleigh-fading channel model is obtained from the Rician-fading channel model by setting \( P_s = 0 \). Once again we assume that the diffuse power in the resolvable paths follows an exponential decay with decay rate \( \mu \). Then \( s^2(\Delta \tau; h) \) is a weighted sum of \( N_{\text{tap}} \) independent central chi-square random variables, each with 2 degrees of freedom. The random variables are identically distributed only if \( \mu = 0 \). The CDF of such a random variable has been derived in closed-form in [3].

### C. Numerical Results

Under the Rician-fading channel model, the asymptotic average probability of detection is largest when \( \Delta \tau = 0 \). Thus the best AROC is a plot of \( \lim_{\alpha \rightarrow 0^+} P_0(\gamma_0(\theta); 0) \) versus \( \delta \). Figs. 1 and 2 show the best AROC for the cases when the diffuse power in all the resolvable paths is equal (\( \mu = 0 \)) and when the diffuse power in the resolvable paths decays at a rate \( \mu = 1 \), respectively. The AROC is plotted for \( M = 128 \), \( N_{\text{tap}} = 1, 2, 4 \) and specular-to-diffuse power ratio 0 and 1 dB. Although the upper bound on the asymptotic average probability of detection is not significant, it is still non-trivial. The AROC is better for the case when there is more power in the specular component. This is due to the fact that a stronger specular component increases the distance between the means of \( s^2(\Delta \tau; h) \) under different hypotheses increasing the separation between the corresponding distributions. For a fixed specular-to-diffuse power ratio, the AROC gets better as the number of resolvable paths increases. This is because the variance of \( s^2(\Delta \tau; h) \) decreases as the diffuse power is distributed over more resolvable paths which has the effect of increasing the separation between the distributions of \( s^2(\Delta \tau; h) \) under different hypotheses. This is also the reason for the AROC being better when \( \mu = 0 \) for a fixed number of resolvable paths.

Figs. 3 and 4 show the best Rayleigh-fading channel AROC for the cases when the diffuse power in all the resolvable paths is equal (\( \mu = 0 \)) and when the diffuse power in the resolvable paths decays at a rate \( \mu = 0.5 \), respectively. The AROC is plotted for \( M = 128 \), \( N_{\text{tap}} = 1, 2, 4 \) and specular-to-diffuse power ratio 0 and 1 dB. Although the upper bound on the asymptotic probability of detection is smaller for the Rayleigh-fading case in comparison to the Rician-fading scenario. Contrary to the Rician case, the AROC gets worse as the number of resolvable paths increases. This is because although the variance of \( s^2(\Delta \tau; h) \) decreases as the number of resolvable paths increases, the decrease in the mean of \( s^2(\Delta \tau; h) \) is more prominent in the absence of the specular component. Hence there is a decrease in the separation between the distributions of \( s^2(\Delta \tau; h) \) under the different hypotheses. For the same reason, the AROC is worse for \( \mu = 0 \) except when there is only one resolvable path.

### V. Acquisition of Ultra-Wideband Signals with Time-Hopping Spreading

The UWB channel is a dense multipath channel without significant fading [10], [11]. In a dense multipath environment, there will be a considerable amount of energy available in the multipath components (MPCs). Considering that we have no information regarding the channel state, there are essentially two ways in which we can attempt to utilize this energy in order to develop a more efficient acquisition scheme. In the first approach, the received signal is first squared to eliminate the channel inversion and then equal gain combining (EGC) is performed to exploit the rich path diversity present in UWB channels. In the second approach, EGC is performed first and the integrator output is then squared to generate the decision statistic. In the sequel, we will refer to the former as square-and-integrate (SAI) and to the latter as integrate-and-square (IAS). In this section, we evaluate and compare the asymptotic acquisition performance of the SAI and IAS approaches for the acquisition of UWB signals with time-hopping spreading.

The presence of a high degree of path diversity in the UWB channel motivates the use of a Rake receiver to improve demodulation performance. The three main Rake receiver structures considered for UWB signal demodulation are the all Rake (ARake), the selective Rake (SRake) and the partial Rake (PRake) receivers [12], [13]. The large number of resolvable multipaths in the UWB channel obviates the use of the ARake receiver due to the complexity involved in its implementation. We assume that the receiver uses a partial Rake (PRake) receiver to perform demodulation. Our choice is guided by the fact that the PRake receiver has lower complexity and still achieves comparable bit error performance relative to the SRake receiver [13].

#### A. System Model

We assume that the propagation channel is modeled by the UWB indoor channel model described in [14]. This model gives a statistical distribution for the path gains based on a UWB propagation experiment but does not address the issue of characterization of the received waveform shape. Due to the frequency sensitivity of the UWB channel, the pulse shapes received at different excess delays are path-dependent [15]. To enable tractable analysis, we assume that the pulse shapes associated with all the propagation paths are identical. The channel impulse response is expressed as

\[
h(t) = \sum_{k=0}^{N_{\text{tap}}-1} p_k h_k f(t - kT_c),
\]

(25)

where \( N_{\text{tap}} \) is the number of taps in the channel response, \( T_c = 2 \) ns is the tap spacing, \( h_k \) is the path gain at excess delay \( kT_c \), \( p_k \) is equally likely to be \( \pm 1 \) to account for signal inversion due to reflections [16] and \( f(t) \) models the combined
effect of the transmitting antenna and the propagation channel on the transmitted pulse. The path gains are independent but not identically distributed with Nakagami-\(m\) distributions. The average energy gains \(\Omega_k = E[h_k^2]\) of the path gains normalized to the total energy received at one meter distance are given by
\[
\Omega_k = \begin{cases} 
\frac{E_{\text{tot}}}{1 + rP(c)}, & \text{for } k = 0 \\
\frac{E_{\text{tot}}}{1 + rP(c)} e^{-(k-1)T_c/\epsilon}, & \text{for } k = 1, 2, \ldots, N_{\text{tap}} - 1,
\end{cases}
\]
where \(E_{\text{tot}}\) is the total average energy in all the paths normalized to the total energy received at one meter distance, \(r\) is the ratio of the average energy of the second MPC and the average energy of the direct path, \(\epsilon\) is the decay constant of the power delay profile and \(F(\epsilon) = \frac{1}{1 - \exp(-kT_c/\epsilon)}\). According to [14], \(E_{\text{tot}}, r\), and \(\epsilon\) are all modeled by lognormal distributions. The Nakagami fading figures \(\{m_k\}\) are distributed according to truncated Gaussian distributions whose mean and variance vary linearly with excess delay. In this paper, these long-term statistics are treated as constants over the duration of the acquisition process.

The transmitted signal is given by
\[
x(t) = \sqrt{P} \sum_{l=\infty}^{\infty} \psi(t - lT_l - c_lT_c), \tag{27}
\]
where \(\psi(t)\) is the UWB monocyte waveform, \(P\) is the transmitted power, \(T_l = N_lT_i\) is the pulse repetition time, \(\{c_l\}\) is the pseudorandom time-hopping (TH) sequence with period \(N_l\), taking integer values between 0 and \(N_l - 1\), and \(T_c\) is the step size of the additional time shift provided by the TH sequence. The pulse repetition time \(T_l\) is chosen to be not less than \((N_l + N_{\text{tap}})T_c\) to avoid overlap between the multipath responses corresponding to distinct transmitted pulses.

If \(u(t) = h(t) \ast x(t)\), the received signal is given by
\[
r(t) = u(t) + n(t)
= \sqrt{E_1} \sum_{l=\infty}^{\infty} w(t - lT_l - c_lT_c - \tau) + n(t), \tag{28}
\]
where
\[
w(t) = \sum_{k=0}^{N_{\text{tap}} - 1} p_k h_k \psi(t - kT_c). \tag{29}
\]
Here \(E_1\) is the total received energy at a distance of one meter from the transmitter, \(\psi_t(t) = f(t) \ast \psi(t)\) is the received UWB pulse of duration \(T_w < T_c\) normalized to have unit energy, \(\tau\) is the propagation delay, and \(n(t)\) is an additive white Gaussian noise (AWGN) process with zero mean and power spectral density \(\sigma^2\).

We assume that the PRake receiver has \(N_p\) fingers where \(N_p \leq N_{\text{tap}}\). Then for true phase \(\tau\), we choose the hit set as \(S_h = \{\tau - (N_l - 1)T_c, \tau - (N_l - 2)T_c, \ldots, \tau + (N_{\text{tap}} - 1)T_c\}\). The phases in the hit set correspond to the phases from which the PRake receiver can collect at least one resolvable path of the channel response corresponding to the true phase. As discussed earlier in Section IV-A, this choice of \(S_h\) corresponds to the best possible asymptotic acquisition performance over all choices of \(S_h\).

B. The Square-and-Integrate (SAI) Approach

In this subsection, we derive the asymptotic performance of an acquisition system which takes the SAI approach.

1) Derivation of the decision statistic: The acquisition system correlates the squared received waveform with a locally generated replica and compares the correlator output to a threshold to determine whether the hypothesized phase of the replica is correct. If the threshold is exceeded, the hypothesized phase becomes the estimate of the true phase. We assume that the normalized received monocyte waveform \(\psi_t(t)\) and the TH sequence \(\{c_l\}\) are known to the receiver. We propose to use an equal gain combiner of window size \(G\). The receiver template signal \(w_t(t)\) is given by
\[
w_t(t) = \sum_{k=0}^{G-1} \psi_t^2(t - kT_c). \tag{30}
\]
The reference TH signal can be obtained by combining the receiver template signal \(w_t(t)\) and the known time hopping sequence as
\[
s(t) = \sum_{l=0}^{MN_l-1} w_t(t - lT_l - c_lT_c - \hat{\tau}), \tag{31}
\]
where \(M\) specifies the number of TH waveform periods in the dwell time and \(\hat{\tau}\) is the hypothesized phase. To simplify the analysis, we assume that the true phase \(\tau\) is an integer multiple of \(T_c\). By the periodicity of the transmitted signal, we have \(0 \leq \tau \leq (N_lN_h - 1)T_c\). The hypothesized phase \(\hat{\tau}\) is also an integer multiple of \(T_c\) with the same range as \(\tau\). Then \(\Delta \tau = \hat{\tau} - \tau = \alpha T_l + \beta T_c\) where \(\alpha\) and \(\beta\) are integers such that \(-N_l + 1 \leq \alpha \leq N_l - 1\) and \(0 \leq \beta \leq N_l - 1\). The correlator output is given by
\[
R(\Delta \tau; h) = \frac{1}{MN_l} \int_{\tau}^{\tau + MN_lT_i} \int_{\tau}^{\tau + MN_lT_i} r(t) s(t) dt \tag{32}
\]
The first term in (32) can be simplified to
\[
s(\Delta \tau; h) = E_1 R_{\psi \psi}(0) \sum_{k=0}^{N_{\text{tap}}-1} r_k(\Delta \tau) h_k^2, \tag{33}
\]
where \(h\) is an \(N_{\text{tap}} \times 1\) vector containing the channel gains, \(R_{\psi \psi}(\nu) = \int_{-\infty}^{\infty} \psi_t^2(t) \psi_t^2(t + \nu) dt\) and \(r_k(\Delta \tau)\), the average number of times the energy in the \(k\)th MPC is collected by one period of the reference TH signal, is given by
\[
r_k(\Delta \tau) = \frac{1}{N_l} \sum_{i=0}^{N_l-1} \sum_{j=0}^{G-1} \chi(c_l + j + \beta, c_l+i+\alpha + k + iN_l), \tag{34}
\]
where \(\chi(a, b) = 1\) if \(a = b\), and 0 otherwise. The value of \(r_k(\Delta \tau)\) depends on the particular pseudo-random TH sequence chosen. To simplify the analysis we assume that
the TH sequence is random and that \( N_{th} \) is large. Under these assumptions, the mean value of \( r_k(\Delta \tau) \) is a reasonable approximation to the actual value. The mean value of \( r_k(\Delta \tau) \) averaged over the set of random TH sequences is given by

\[
E[r_k(\alpha T_t + \beta T_{\hat{c}})] = U(\beta + G - 1, k + i_1 N_t)U(k + i_1 N_t, \beta) + \sum_{i=0}^{G-1} \min\{j+\beta,k+i_1 N_t\} - 1 N_{th},
\]

where the second equality is obtained by exploiting the similarity between the integral in (36) and the first term of (32). We have also used the fact that

\[
s^2(t) = \sum_{k=0}^{N_{th}-1} \sum_{l=0}^{G-1} \psi_l(t - kT_c - lT_l - cT_c - \delta),
\]

which differs from (31) only in the exponent of the received pulse waveform \( \psi_l(t) \).

We approximate the third term in (32) by a Gaussian random variable with mean \( \mu_y \) and variance \( \nu_y^2 \) which are given by

\[
\mu_y = \frac{1}{M N_{th}} \int_{0}^{\tau} s(t)dt,
\]

\[
\nu_y^2 = \frac{1}{M^2 N_{th}^2} \int_{0}^{\tau} s^2(t)dt - \mu_y^2.
\]

respectively. Note that the expectation in the derivation of \( \mu_y \) and \( \nu_y^2 \) is only with respect to the noise process \( n(t) \). This approximation is accurate provided that the product of the integration time \( M N_{th} T_l \) and the bandwidth of the system \( B \) is large [2, pp. 240–250], which is the case for the scenarios we consider.

Then the correlator output can be written as

\[
R(\Delta \tau; h) = s(\Delta \tau; h) + n_y,
\]

where, conditioned on \( h \), \( n_y \) is a Gaussian random variable with mean \( \mu_y \) and variance \( \nu_y^2 \). The probabilities of false alarm and detection conditioned on the particular channel realization and given the decision threshold \( \gamma \) are given as

\[
P_{FA}(\gamma, \Delta \tau | h) = P_N(\gamma, \Delta \tau | h), \hat{\tau} \notin S_h, \quad Q \left( \frac{\gamma - s(\Delta \tau; h) - \mu_y}{\nu_y} \right), \hat{\tau} \notin S_h,
\]

\[
P_{D}(\gamma, \Delta \tau | h) = P_N(\gamma, \Delta \tau | h), \hat{\tau} \in S_h, \quad Q \left( \frac{\gamma - s(\Delta \tau; h) - \mu_y}{\nu_y} \right), \hat{\tau} \in S_h.
\]

2) Asymptotic Performance of the SAI Approach: Once again, we need to verify that the conditions of Theorem 1 hold for the acquisition system described above, before we can apply it. Henceforth, the arguments are very similar to those made in Section IV-B. Since the path gains are distributed according to Nakagami-\( m \) distributions, \( h \) has an absolutely continuous distribution and hence \( s(\Delta \tau; h) \) has an absolutely continuous distribution. Since \( S_h \) is finite, for any threshold \( \gamma \geq 0 \) and every \( \epsilon > 0 \), there exists a \( \kappa(\gamma, \epsilon) > 0 \) such that for all \( \hat{\tau} \in S_p \) we have

\[
Pr(\{ h : s(\Delta \tau; h) \leq \gamma + \kappa(\gamma, \epsilon) \}) < \frac{\epsilon}{2}.
\]

Note that \( s(\Delta \tau; h) \) is a non-negative random variable for all \( \hat{\tau} \in S_p \). Then by choosing a positive integer \( n \) such that \( n^{-1} < \epsilon/2 \) and a positive real number \( K_s \geq \max\{\text{mean}(s(\Delta \tau; h)) : \hat{\tau} \in S_p\} \), for all \( \hat{\tau} \in S_p \) we get

\[
Pr(\{ h : s(\Delta \tau; h) \geq n K_s \}) \leq \frac{\text{mean}(s(\Delta \tau; h))}{n K_s} \leq \frac{1}{n} < \frac{\epsilon}{2}.
\]

In Appendix C, we show that \( A_{\delta}(\gamma; \Delta \tau) \) and \( B_{\delta}(\gamma; \Delta \tau) \) defined below satisfy the conditions of Theorem 1.

\[
A_{\delta}(\gamma; \Delta \tau) = \{ h : s(\Delta \tau; h) < \gamma - \kappa(\gamma, \epsilon) \},
\]

\[
B_{\delta}(\gamma; \Delta \tau) = \{ h : \gamma + \kappa(\gamma, \epsilon) < s(\Delta \tau; h) < n K_s \}.
\]

Using arguments very similar to those used in Section IV-B, we can show that

\[
\lim_{\epsilon \to 0^+} Pr(A_{\epsilon}(\gamma; \Delta \tau)) = Pr(\{ h : s(\Delta \tau; h) \leq \gamma \}),
\]

\[
\lim_{\epsilon \to 0^+} Pr(B_{\epsilon}(\gamma; \Delta \tau)) = Pr(\{ h : s(\Delta \tau; h) \geq \gamma \}).
\]

Then by Theorem 1, for any \( \delta > 0 \) if there exists a threshold \( \gamma \) and an \( \eta_{\gamma}(\delta) > 0 \) such that \( P_{FA}(\gamma, \Delta \tau) < \delta \) for all \( \sigma^2 < \eta_{\gamma}(\delta) \) and for all \( \hat{\tau} \notin S_h \), then

\[
\lim_{\sigma \to 0^+} P_M(\gamma; \Delta \tau) \geq Pr(\{ h : s(\Delta \tau; h) \leq \gamma_m(\delta) \}(48))
\]
where $\gamma_m(\delta) = \inf\{\gamma : \Pr(\{h : s(\Delta r; h) \geq \gamma\}) \leq \delta, \text{ for all } \hat{r} \notin S_h}\). Finally, we have the following upper bound on the asymptotic average probability of detection,

$$
\lim_{\sigma \to 0^+} P_D(\gamma; \Delta r) \leq \Pr(\{h : s(\Delta r; h) \geq \gamma_m(\delta)\}),
$$

where $\hat{r} \in S_h$. Once again, we observe that the AROC for a particular $\hat{r} \in S_h$ depends on the separation between the corresponding distribution of $s(\Delta r; h)$ and the distributions of $s(\Delta r; h)$ for all $\hat{r} \notin S_h$. The CDF of $s(\Delta r; h)$ is needed to calculate the AROC. From (33), $s(\Delta r; h)$ is a linear combination of independent random variables and hence its characteristic function is given by

$$
\hat{\phi}_s(\omega; \Delta r) = \prod_{k=0}^{N_{tap}-1} \phi_k(E_1 R \psi_k(0)r_k(\Delta r)\omega),
$$

where $\phi_k(\cdot)$'s are the characteristic functions of the Gamma distributed $h_k^2$'s [17]. The CDF of $s(\Delta r; h)$ is obtained by substituting (50) in (24).

### C. The Integrate-and-Square (IAS) Approach

In this subsection, we derive the asymptotic performance of an acquisition system which takes the IAS approach. The derivation of the decision statistic in this case is very similar to the decision statistic derivation in the previous section. All the relevant assumptions made in the previous section, to enable tractable analysis, still hold unless stated otherwise. To avoid repetition, we only define those quantities which have not already been defined in the previous subsection.

1) Derivation of the decision statistic: In this approach, the acquisition system correlates the received waveform with a locally generated template signal and squares the integrator output to generate the decision statistic. The receiver template signal $v_r(t)$ is given by

$$
v_r(t) = \sum_{k=0}^{G-1} v(t - kT_c).
$$

The reference TH signal is given by

$$
q(t) = \sum_{l=0}^{MN_{th}-1} v(t - lT_l - c_lT_c - \hat{r}).
$$

The decision statistic is given by

$$
R(\Delta r; h) = \frac{1}{MN_{th}} \left[ \frac{r(t)q(t)dt}{\sqrt{E_1} \sum_{k=0}^{N_{tap}-1} r_k(\Delta r)p_k h_k + n_x} \right]^2
$$

where $n_x$ is a zero-mean Gaussian random variable with variance $\sigma_x^2 = \frac{G^2 \sigma^2}{MN_{th}}$ and $r_k(\Delta r)$ is given in (34). The probabilities of false alarm and detection conditioned on the particular channel realization and given the decision threshold $\gamma \geq 0$ are given by

$$
\begin{align*}
P_{FA}(\gamma; \Delta r|\hat{r}) &= F_N'(\gamma; \Delta r|\hat{r}), \hat{r} \notin S_h, \\
&= Q\left(\frac{\sqrt{r - s(\Delta r; h)}}{\sigma_z}\right) + Q\left(\frac{\sqrt{r + s(\Delta r; h)}}{\sigma_z}\right), \hat{r} \notin S_h, \\
&= Q\left(\frac{\sqrt{r - s(\Delta r; h)}}{\sigma_z}\right) + Q\left(\frac{\sqrt{r + s(\Delta r; h)}}{\sigma_z}\right), \hat{r} \in S_h.
\end{align*}
$$

2) Asymptotic Performance of the IAS Approach: As before, $s(\Delta r; h)$ has an absolutely continuous distribution and hence for any threshold $\gamma \geq 0$ and every $\epsilon > 0$, there exists a $\kappa(\gamma, \epsilon) > 0$ such that for all $\hat{r} \in S_p$, we have

$$
\begin{align*}
\Pr(\{h : |s(\Delta r; h) - \sqrt{\gamma}| \leq \kappa(\gamma, \epsilon) &\text{ or } |s(\Delta r; h) + \sqrt{\gamma}| \leq \kappa(\gamma, \epsilon)\}) < \epsilon. \\
\end{align*}
$$

In Appendix D, we show that $A_c(\gamma; \Delta r)$ and $B_c(\gamma; \Delta r)$ defined below satisfy the conditions of Theorem 1.

$$
\begin{align*}
A_c(\gamma; \Delta r) &= \{h : -\sqrt{\gamma} + \kappa(\gamma, \epsilon) < s(\Delta r; h) < \sqrt{\gamma} - \kappa(\gamma, \epsilon)\}, \\
B_c(\gamma; \Delta r) &= \{h : s(\Delta r; h) > \sqrt{\gamma} + \kappa(\gamma, \epsilon) &\text{ or } s(\Delta r; h) < -\sqrt{\gamma} - \kappa(\gamma, \epsilon)\}. \\
\end{align*}
$$

We can also show that

$$
\begin{align*}
\lim_{\epsilon \to 0^+} \Pr(A_c(\gamma; \Delta r)) &= \Pr(\{h : |s(\Delta r; h) - \sqrt{\gamma}| \leq \kappa(\gamma, \epsilon)\}), \\
\lim_{\epsilon \to 0^+} \Pr(B_c(\gamma; \Delta r)) &= \Pr(\{h : s(\Delta r; h) \geq \sqrt{\gamma} \text{ or } s(\Delta r; h) \leq -\sqrt{\gamma}\}) \\
&= \Pr(\{h : s(\Delta r; h) \geq \sqrt{\gamma} \text{ or } s(\Delta r; h) \leq -\sqrt{\gamma}\}).
\end{align*}
$$

Then by Theorem 1, for any $\delta > 0$ if there exists a threshold $\gamma$ and an $\eta_1(\delta) > 0$ such that $F_{FA}(\gamma; \Delta r) < \delta$ for all $\sigma^2 < \eta_1(\delta)$ and for all $\hat{r} \notin S_h$, then

$$
\lim_{\sigma \to 0^+} P_M(\gamma; \Delta r)
\geq \Pr(\{h : |s(\Delta r; h) - \sqrt{\gamma_m(\delta)}| \leq \sqrt{\gamma_m(\delta)}\})
$$

where $\gamma_m(\delta) = \inf\{\gamma : \Pr(\{h : s(\Delta r; h) - \sqrt{\gamma} \text{ or } s(\Delta r; h) \leq -\sqrt{\gamma}\}) \leq \delta, \text{ for all } \hat{r} \notin S_h\}$. Finally, we have the following upper bound on the asymptotic average probability of detection.

$$
\begin{align*}
\lim_{\sigma \to 0^+} P_D(\gamma; \Delta r) &\leq \Pr(\{h : s(\Delta r; h) \geq \sqrt{\gamma_m(\delta)} \text{ or } s(\Delta r; h) \leq -\sqrt{\gamma_m(\delta)}\}), \\
&= \phi_k(\sqrt{E_1}r_k(\Delta r)\omega) + \phi_k(-\sqrt{E_1}r_k(\Delta r)\omega). \\
\end{align*}
$$

where $\hat{r} \in S_h$. Once again, the AROC calculation requires the CDF of $s(\Delta r; h)$. Since the polarities $p_k$ and path gains $h_k$ are independent, the characteristic function of $s(\Delta r; h)$, in this case, is given by

$$
\phi_s(\omega; \Delta r) = \prod_{k=0}^{N_{tap}-1} \left[ \phi_k(\sqrt{E_1}r_k(\Delta r)\omega) + \phi_k(-\sqrt{E_1}r_k(\Delta r)\omega) \right].
$$
where \(\phi_k(\cdot)\) is the characteristic function of the Nakagami-\(m\) distributed \(h_k\) [17]. Substitution of the above equation in (24) yields the CDF of \(s(\Delta\tau; h)\).

D. Numerical Results

To calculate the AROC for the SAI and IAS acquisition schemes, we choose the following values for the system parameters: the TH sequence period \(N_{th} = 1024\), \(N_0 = 16\), \(M = 1\), the length of the channel response \(N_{tap} = 100\), the number of PRake fingers \(N_p = 5\) and \(N_t = 116\). We assume that \(E_{tot} = -20.4\) dB which is its mean value when the transmitter-receiver (T-R) separation is 10 m [14]. We choose the power ratio \(r = -4\) dB, decay constant \(\epsilon = 16.1\) dB and fading figures \(m_k = 3.5 - \frac{kN_0}{4}\), \(0 \leq k \leq N_{tap} - 1\), which are their mean values given in [14]. The best AROC, which is again a plot of \(\lim_{\tau \to -\infty} P_D(\gamma_m(\delta); 0)\) versus \(\delta\), does not depend on the received power and hence we set \(E_1 = 1\).

Fig. 5 shows the best AROC of the SAI approach for EGC window sizes \(G = 1, 2, 5, 10\) and 15. The AROC is worse in comparison to the AROC for the DS-SS system for all the EGC window sizes considered. For instance, the upper bound on the asymptotic probability of detection is at best 0.94 when \(\delta = 0.05\) for the SAI approach, but for the DS-SS system it is at least 0.95 when \(\delta = 0.05\). The AROC becomes worse as the EGC window length increases and is best for \(G = 1\), which is equivalent to the case when there is no EGC. This does not necessarily imply that performing EGC for acquisition is not advantageous since the AROC is just a characterization of the acquisition performance achievable at large SNRs which may not capture the effect of EGC at low SNRs. As \(G\) increases the signal energy collected by the EGC window \(s(\Delta\tau; h)\) increases both when \(\tau = \hat{\tau}\) and \(\tau \notin S_h\). For \(\hat{\tau} = \tau\), the additional energy collected is from the non-LOS paths which are weaker in comparison to the LOS path and thus the increase in signal energy is relatively small. The increase is more significant when \(\hat{\tau} \notin S_h\) since the additional energy is comparable to the energy collected when \(G = 1\). Thus the separation between the distributions of \(s(\Delta\tau; h)\) when \(\hat{\tau} = \tau\) and \(\hat{\tau} \notin S_h\) decreases, causing the AROC to get worse.

Fig. 6 shows the best AROC of the IAS approach for EGC window sizes \(G = 1, 2, 5, 10\) and 15. The upper bound on the asymptotic average probability of detection is almost trivial for \(G = 1\) and becomes significantly restrictive as \(G\) increases. As \(G\) increases, for \(\tau = \hat{\tau}\) the EGC window collects multiple paths which may have opposing polarities resulting in cancellations and hence a decrease in the probability of detection. For \(G = 1\), this cancellation is absent when \(\hat{\tau} = \tau\) but still occurs when \(\hat{\tau} \notin S_h\) since the random time-hopping sequence facilitates collection of multiple paths. Thus the signal energy collected when \(\hat{\tau} \notin S_h\) is much smaller than the signal energy collected when \(\hat{\tau} = \tau\), resulting in a significant separation between the corresponding distributions of \(s(\Delta\tau; h)\). Hence the AROC is not restrictive for \(G = 1\). Since the best AROC is just an upper bound on the AROCs of all the hit set phases, we plot for \(G = 1\) the AROCs of the phases \(\hat{\tau} \in S_h\) corresponding to \(\Delta\tau = 5T_c, 10T_c, 15T_c, 20T_c\) and \(30T_c\) in Fig. 7. We see that even for IAS with \(G = 1\) the bound on the asymptotic average probability of detection becomes increasingly restrictive as the distance of the hit set phase from the LOS path increases. This is because the energy in the paths decays with increase in distance from the LOS path.

VI. Conclusions

A typical timing acquisition system consists of a verification stage in which a threshold crossing at a candidate phase is checked to see if it was a false alarm or a true detection event. The usual procedure for implementing the verification stage is to have a large dwell time for the correlator [2]. The large dwell time increases the effective SNR of the decision statistic and in the absence of channel fading, this results in accurate verification. In this paper, we evaluated the asymptotic performance of threshold-based timing acquisition systems in the presence of multipath fading and found that, no matter how large the SNR is or how we choose the threshold, there are fading scenarios in which false alarms and misses occur with non-zero and sometimes significant average probability. Thus it may not be possible to build a good verification stage for threshold-based acquisition systems operating in such channels by just increasing the dwell time. We found that if we choose a threshold such that the average probability of false alarm is less than a given tolerance, then there is a possibly non-trivial lower bound on the asymptotic average probability of miss. This lower bound translates to an upper bound on the asymptotic average probability of detection. We evaluated this upper bound for a threshold-based DS-SS signal acquisition system operating in Rician- and Rayleigh-fading channels and found that it is non-trivial, though not very significant. We also evaluated this upper bound for two threshold-based approaches, namely SAI and IAS, for the acquisition of UWB signals with time-hopping spreading. For SAI, we found that the upper bound on the asymptotic average probability of detection was significantly restrictive for all values of EGC window size. But for IAS, the upper bound was almost trivial at least for some hit set phases when there was no EGC being done. Nevertheless, there were still some hit set phases where the upper bound was restrictive. These results seem to suggest that EGC may not be a good strategy to improve acquisition performance. More importantly, they suggest that acquisition might be a potential bottleneck on throughput in any UWB-based packet network employing threshold-based acquisition systems.

APPENDIX

A. Proof of Theorem 1

By the hypothesis, for every \(\gamma\) and \(\epsilon > 0\) there is an \(\eta(\gamma, \epsilon) > 0\) such that when \(\sigma^2 < \eta(\gamma, \epsilon)\) there exists a subset \(A_\epsilon(\gamma; \Delta\tau)\) of \(\mathcal{H}\) such that for all \(h \in A_\epsilon(\gamma; \Delta\tau)\), \(F_N(\gamma; \Delta\tau|h) > 1 - \epsilon\). Then for all \(\sigma^2 < \eta(\gamma, \epsilon)\), we have

\[
P_M(\gamma; \Delta\tau) \geq E_H[F_N(\gamma; \Delta\tau|h)I_{A_\epsilon(\gamma; \Delta\tau)}(h)] \geq (1 - \epsilon) \Pr(A_\epsilon(\gamma; \Delta\tau)) \geq \Pr(A_\epsilon(\gamma; \Delta\tau)) - \epsilon,
\]

(59)
where \( I_{A_e}(\gamma;\Delta \tau)(\cdot) \) is the indicator function of the set \( A_e(\gamma;\Delta \tau) \). Furthermore, when \( \sigma^2 < \eta(\gamma, \epsilon) \) and \( D_e(\gamma;\Delta \tau) = A_e(\gamma;\Delta \tau) \cup B_e(\gamma;\Delta \tau) \) we have

\[
P_M(\gamma;\Delta \tau) \leq E_H[F_N(\gamma;\Delta \tau|h)(I_{D_e}(\gamma;\Delta \tau) + I_{D_e^2}(\gamma;\Delta \tau)(h))] \\
\leq E_H[F_N(\gamma;\Delta \tau|h)(I_{A_e}(\gamma;\Delta \tau) + I_{B_e}(\gamma;\Delta \tau) + I_{D_e^2}(\gamma;\Delta \tau)(h))] \\
\leq \Pr(A_e(\gamma;\Delta \tau)) + \epsilon \Pr(B_e(\gamma;\Delta \tau)) + \epsilon \\
\leq \Pr(A_e(\gamma;\Delta \tau)) + 2\epsilon. \quad (60)
\]

Consider any convergent positive sequence \( \{\sigma_n\} \) with limit zero. For any \( \epsilon > 0 \), from (59) and (60), we have

\[
\Pr(A_e(\gamma;\Delta \tau)) - \epsilon \leq \liminf_{\sigma_n \to 0^+} P_M(\gamma;\Delta \tau) \\
\leq \limsup_{\sigma_n \to 0^+} P_M(\gamma;\Delta \tau) \\
\leq \Pr(A_e(\gamma;\Delta \tau)) + 2\epsilon. \quad (61)
\]

Now consider a convergent positive sequence \( \{\epsilon_n\} \) with limit zero. Since (61) holds for every \( \epsilon > 0 \), we have

\[
\limsup_{\epsilon_n \to 0^+} \Pr(A_{\epsilon_n}(\gamma;\Delta \tau)) = \limsup_{\epsilon_n \to 0^+} \left[ \Pr(A_{\epsilon_n}(\gamma;\Delta \tau)) - \epsilon_n \right] \\
\leq \liminf_{\epsilon_n \to 0^+} P_M(\gamma;\Delta \tau) \\
\leq \limsup_{\epsilon_n \to 0^+} P_M(\gamma;\Delta \tau) \\
\leq \liminf_{\epsilon_n \to 0^+} \Pr(A_{\epsilon_n}(\gamma;\Delta \tau)) + 2\epsilon_n \\
= \liminf_{\epsilon_n \to 0^+} \Pr(A_{\epsilon_n}(\gamma;\Delta \tau)). \quad (62)
\]

But for any sequence \( \{\epsilon_n\} \),

\[
\liminf_{\epsilon_n \to 0^+} \Pr(A_{\epsilon_n}(\gamma;\Delta \tau)) \leq \limsup_{\epsilon_n \to 0^+} \Pr(A_{\epsilon_n}(\gamma;\Delta \tau)). \quad (63)
\]

So we have

\[
\liminf_{\epsilon_n \to 0^+} \Pr(A_{\epsilon_n}(\gamma;\Delta \tau)) = \limsup_{\epsilon_n \to 0^+} \Pr(A_{\epsilon_n}(\gamma;\Delta \tau)). \quad (64)
\]

Thus \( \lim_{\epsilon_n \to 0^+} \Pr(A_{\epsilon_n}(\gamma;\Delta \tau)) \) exists for every positive sequence \( \{\epsilon_n\} \) converging to zero. Furthermore, all the inequalities in (62) are actually equalities and \( \lim_{\epsilon_n \to 0^+} P_M(\gamma;\Delta \tau) \) exists for all sequences \( \{\sigma_n\} \). By fixing the sequence \( \{\epsilon_n\} \) and considering all possible positive sequences \( \{\sigma_n\} \) converging to zero, we see that \( \lim_{\sigma_n \to 0^+} P_M(\gamma;\Delta \tau) = \lim_{\epsilon_n \to 0^+} \Pr(A_{\epsilon_n}(\gamma;\Delta \tau)) \) for all sequences \( \{\sigma_n\} \). Thus by the definition of the limit of a function [18] we have

\[
\lim_{\sigma \to 0^+} P_M(\gamma;\Delta \tau) = \lim_{\epsilon_n \to 0^+} \Pr(A_{\epsilon_n}(\gamma;\Delta \tau)). \quad (65)
\]

Since the left hand side in (65) is fixed for all sequences \( \{\epsilon_n\} \), by the definition of the limit of a function we have

\[
\lim_{\sigma \to 0^+} P_M(\gamma;\Delta \tau) = \lim_{\epsilon \to 0^+} \Pr(A_e(\gamma;\Delta \tau)). \quad (66)
\]

Similarly, for \( \sigma^2 < \eta(\gamma, \epsilon) \) we have

\[
\Pr(B_e(\gamma;\Delta \tau)) - \epsilon < P_{FA}(\gamma;\Delta \tau) < \Pr(B_e(\gamma;\Delta \tau)) + 2\epsilon \quad (67)
\]

and hence we can show that

\[
\lim_{\sigma \to 0^+} P_{FA}(\gamma;\Delta \tau) = \lim_{\epsilon \to 0^+} \Pr(B_e(\gamma;\Delta \tau)). \quad (68)
\]

Let \( \gamma^* \) be a threshold such that \( P_{FA}(\gamma^*;\Delta \tau) < \delta \) for all \( \sigma^2 < \eta_1(\delta) \) and \( \tilde{\tau} \notin S_h \). Since the complementary conditional CDF \( F_N(\gamma;\Delta \tau|h) \) is a non-increasing function of the threshold \( \gamma \), the average probability of false alarm \( P_{FA}(\gamma;\Delta \tau) \) is a non-increasing function of \( \gamma \). For all \( \sigma^2 < \eta_1(\delta) \) we have

\[
\gamma^* \geq \inf\{\gamma : P_{FA}(\gamma;\Delta \tau) < \delta, \text{ for all } \tilde{\tau} \notin S_h\}, \quad (69)
\]

Since (69) holds for all \( \sigma^2 < \eta_1(\delta) \), we have

\[
\gamma^* \geq \inf\{\gamma : \lim_{\sigma \to 0^+} P_{FA}(\gamma;\Delta \tau) \leq \delta, \text{ for all } \tilde{\tau} \notin S_h\} = \inf\{\gamma : \lim_{\epsilon \to 0^+} \Pr(B_e(\gamma;\Delta \tau)) \leq \delta, \text{ for all } \tilde{\tau} \notin S_h\}, \quad (70)
\]

where the equality follows from (68). Note that the expression on the right hand side of the equality in (70) is equal to \( \gamma_m(\delta) \) defined in the statement of the theorem.

Since \( \gamma^* \geq \gamma_m(\delta) \), for \( \tilde{\tau} \in S_h \) we have

\[
P_M(\gamma^*;\Delta \tau) = E_H[F_N(\gamma^*;\Delta \tau|h)] \geq E_H[F_N(\gamma_m(\delta);\Delta \tau|h)] = P_M(\gamma_m(\delta);\Delta \tau), \quad (71)
\]

where the inequality follows from the fact that \( F_N(\gamma;\Delta \tau|h) \) being a conditional CDF is an increasing function of \( \gamma \). From (66) and (71), we have

\[
\lim_{\sigma \to 0^+} P_M(\gamma^*;\Delta \tau) \geq \lim_{\epsilon \to 0^+} \Pr(A_e(\gamma_m(\delta);\Delta \tau)), \quad (72)
\]

for all \( \tilde{\tau} \in S_h \), which proves the first statement of the theorem.

From (66), given \( \xi > 0 \) and a threshold \( \gamma \), there exists a \( \eta_2(\gamma, \xi) > 0 \) such that

\[
P_M(\gamma;\Delta \tau) \geq \lim_{\epsilon \to 0^+} \Pr(A_e(\gamma;\Delta \tau)) - \xi \quad (73)
\]

for all \( \sigma^2 < \eta_2(\gamma, \xi) \). Then from (71) and (73), we have

\[
P_M(\gamma^*;\Delta \tau) \geq P_M(\gamma_m(\delta);\Delta \tau) \geq \lim_{\epsilon \to 0^+} \Pr(A_e(\gamma_m(\delta);\Delta \tau)) - \xi \quad (74)
\]

for all \( \sigma^2 < \eta_2(\gamma_m(\delta), \xi) = \kappa(\delta, \xi) \). This proves the second statement of the theorem.

**B. Proof that \( A_e(\gamma;\Delta \tau) \) and \( B_e(\gamma;\Delta \tau) \) defined in (14) satisfy the conditions of Theorem 1**

The first condition on the sets is verified in the following manner.

\[
\Pr(A_e(\gamma;\Delta \tau) \cup B_e(\gamma;\Delta \tau)) \\
= 1 - \Pr(A_e^c(\gamma;\Delta \tau) \cap B_e^c(\gamma;\Delta \tau)) \\
= 1 - \Pr\{(h : \sqrt{\gamma} - \kappa(\gamma, \epsilon) \leq s(\Delta \tau; h) \leq \sqrt{\gamma} + \kappa(\gamma, \epsilon))\} > 1 - \epsilon. \quad (75)
\]

To verify the other conditions of Theorem 1, we need the following properties of the generalized Marcum’s Q-function [17]. For \( \beta > \alpha \geq 0 \),

\[
Q_1(\alpha, \beta) \leq \exp \left[ -\frac{(\beta - \alpha)^2}{2} \right], \quad (76)
\]
and for $\alpha > \beta \geq 0$,

$$Q_1(\alpha, \beta) \geq 1 - \frac{1}{2} \exp \left[ - \frac{(\beta - \alpha)^2}{2} \right] + \frac{1}{2} \exp \left[ - \frac{(\beta + \alpha)^2}{2} \right] \geq 1 - \exp \left[ - \frac{(\beta - \alpha)^2}{2} \right]. \quad (77)$$

Furthermore for $\sigma^2 < \eta(\gamma, \epsilon) = \frac{2^{2(\gamma, \epsilon)}}{2 \ln 2}$ and $h \in \mathcal{A}_n(\gamma; \Delta \tau)$,

$$F_N(\gamma; \Delta \tau| h) = 1 - Q_1 \left( \frac{s(\Delta \tau; h)}{\sigma}, \frac{\sqrt{\gamma}}{\sigma} \right) \geq 1 - \exp \left[ - \frac{(\sqrt{\gamma} - s(\Delta \tau; h))^2}{2\sigma^2} \right] \geq 1 - \frac{\kappa^2(\gamma, \epsilon)}{2\sigma^2} \geq 1 - \frac{\kappa^2(\gamma, \epsilon)}{2\eta(\gamma, \epsilon)} = 1 - \epsilon, \quad (78)$$

and for $h \in \mathcal{B}_n(\gamma; \Delta \tau)$,

$$F_N(\gamma; \Delta \tau| h) = 1 - Q_1 \left( \frac{s(\Delta \tau; h)}{\sigma}, \frac{\sqrt{\gamma}}{\sigma} \right) \leq \exp \left[ - \frac{(\sqrt{\gamma} - s(\Delta \tau; h))^2}{2\sigma^2} \right] \leq \exp \left[ - \frac{\kappa^2(\gamma, \epsilon)}{2\sigma^2} \right] \leq \exp \left[ - \frac{\kappa^2(\gamma, \epsilon)}{2\eta(\gamma, \epsilon)} \right] = \epsilon. \quad (79)$$

C. Proof that $\mathcal{A}_n(\gamma; \Delta \tau)$ and $\mathcal{B}_n(\gamma; \Delta \tau)$ defined in (46) satisfy the conditions of Theorem 1

The first condition on the sets is verified in the following manner using (44) and (45).

$$\Pr(\mathcal{A}_n(\gamma; \Delta \tau)) = 1 - \Pr(\mathcal{A}_n^c(\gamma; \Delta \tau)) \leq 1 - \Pr(\{h : |s(\Delta \tau; h) - \gamma| \leq \kappa(\gamma, \epsilon) \text{ or } s(\Delta \tau; h) \geq nK_s \}) \geq 1 - \frac{\epsilon}{2} - \frac{\epsilon}{2} = 1 - \epsilon. \quad (81)$$

From (33) and (34), we see that $\sigma^2(\Delta \tau; h) = \sigma^2 K^2_s s(\Delta \tau; h)$ where $K_s = \frac{4R_{(0,0)}(\Delta \tau)}{\sigma^2 s(\Delta \tau; h)}$. Then for $h \in \mathcal{A}_n(\gamma; \Delta \tau)$, we have

$$F_N(\gamma; \Delta \tau| h) = 1 - Q \left( \frac{\gamma - s(\Delta \tau; h) - \mu_y}{\sqrt{\sigma^2 K^2_s s(\Delta \tau; h) + \nu^2_y}} \right) \geq 1 - Q \left( \frac{\kappa(\gamma, \epsilon) - \mu_y}{\sqrt{\sigma^2 K^2_s (\gamma - \kappa(\gamma, \epsilon)) + \nu^2_y}} \right).$$

We can assume, without loss of generality, that $\gamma - \kappa(\gamma, \epsilon) \geq 0$. Then the $Q$-function in the above equation is a decreasing function of $\sigma^2$ and converges to 0 as $\sigma^2 \to 0$. So there exists an $\eta_1(\gamma, \epsilon) > 0$ such that $F_N(\gamma; \Delta \tau| h) > 1 - \epsilon$ for all $h \in \mathcal{A}_n(\gamma; \Delta \tau)$ whenever $\sigma^2 < \eta_1(\gamma, \epsilon)$. For $h \in \mathcal{B}_n(\gamma; \Delta \tau)$, we have

$$F_N(\gamma; \Delta \tau| h) = 1 - Q \left( \frac{\gamma - s(\Delta \tau; h) - \mu_y}{\sqrt{\sigma^2 K^2_s s(\Delta \tau; h) + \nu^2_y}} \right) \geq Q \left( \frac{\mu_y + s(\Delta \tau; h) - \gamma}{\sqrt{\sigma^2 K^2_s s(\Delta \tau; h) + \nu^2_y}} \right) \leq \frac{\kappa(\gamma, \epsilon) + \mu_y}{\nu^2_y} \frac{1}{\sigma^2 K^2_s K_s + \nu^2_y}. \quad (82)$$

As before, the $Q$-function is a decreasing function of $\sigma^2$ and hence there exists an $\eta_2(\gamma, \epsilon) > 0$ such that $F_N(\gamma; \Delta \tau| h) < \epsilon$ for all $h \in \mathcal{B}_n(\gamma; \Delta \tau)$ whenever $\sigma^2 < \eta_2(\gamma, \epsilon)$. By choosing $\eta(\gamma, \epsilon) = \min(\eta_1(\gamma, \epsilon), \eta_2(\gamma, \epsilon))$, we see that the second and third conditions of Theorem 1 hold for $\sigma^2 < \eta(\gamma, \epsilon)$.

D. Proof that $\mathcal{A}_n(\gamma; \Delta \tau)$ and $\mathcal{B}_n(\gamma; \Delta \tau)$ defined in (55) satisfy the conditions of Theorem 1

The first condition on the sets is verified in the following manner using (54).

$$\Pr(\mathcal{A}_n(\gamma; \Delta \tau) \cup \mathcal{B}_n(\gamma; \Delta \tau)) = 1 - \Pr(\mathcal{A}_n^c(\gamma; \Delta \tau) \cap \mathcal{B}_n^c(\gamma; \Delta \tau)) = 1 - \Pr(\{h : |s(\Delta \tau; h) - \sqrt{\gamma}| \leq \kappa(\gamma, \epsilon) \text{ or } s(\Delta \tau; h) + \sqrt{\gamma} \leq \kappa(\gamma, \epsilon)\}) \geq 1 - \epsilon. \quad (83)$$

Furthermore for $\sigma^2 < \eta(\gamma, \epsilon) = \frac{2^{2(\gamma, \epsilon)}}{2 \ln 2}$ and $h \in \mathcal{A}_n(\gamma; \Delta \tau)$,

$$F_N(\gamma; \Delta \tau| h) = 1 - Q \left( \frac{\sqrt{\gamma} - s(\Delta \tau; h)}{\sigma} \right) - Q \left( \frac{\sqrt{\gamma} + s(\Delta \tau; h)}{\sigma} \right) \geq 1 - \frac{1}{2} \exp \left[ - \frac{(\sqrt{\gamma} - s(\Delta \tau; h))^2}{2\sigma^2} \right] - \frac{1}{2} \exp \left[ - \frac{(\sqrt{\gamma} + s(\Delta \tau; h))^2}{2\sigma^2} \right] \geq 1 - \frac{\kappa^2(\gamma, \epsilon)}{2\sigma^2} \geq 1 - \epsilon. \quad (84)$$

For $h \in \mathcal{B}_n(\gamma; \Delta \tau)$ such that $s(\Delta \tau; h) > \sqrt{\gamma} + \kappa(\gamma, \epsilon)$,

$$F_N(\gamma; \Delta \tau| h) = 1 - Q \left( \frac{\sqrt{\gamma} - s(\Delta \tau; h)}{\sigma} \right) - Q \left( \frac{\sqrt{\gamma} + s(\Delta \tau; h)}{\sigma} \right) = Q \left( \frac{\kappa(\gamma, \epsilon) - s(\Delta \tau; h)}{\sigma} \right) - Q \left( \frac{\sqrt{\gamma} + s(\Delta \tau; h)}{\sigma} \right) \leq \exp \left[ - \frac{(s(\Delta \tau; h) - \sqrt{\gamma})^2}{2\sigma^2} \right] < \frac{\kappa^2(\gamma, \epsilon)}{2\sigma^2} \leq \frac{\epsilon}{2}. \quad (85)$$
Similarly for $h \in B_{s}(\gamma; \Delta \tau)$ such that $s(\Delta \tau; h) < -\sqrt{\gamma} - k(\gamma, \epsilon)$, we can show that $F_{N}(\gamma; \Delta \tau| h) < \frac{1}{2}$.

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Fig. 1. The best AROC for the Rician-fading channel model with diffuse power decay rate $\mu = 0$.

Fig. 2. The best AROC for the Rician-fading channel model with diffuse power decay rate $\mu = 1$.

Fig. 3. The best AROC for the Rayleigh-fading channel model with diffuse power decay rate $\mu = 0$.

Fig. 4. The best AROC for the Rayleigh-fading channel model with diffuse power decay rate $\mu = 0.5$. 
Fig. 5. The best AROC of the SAI approach to UWB signal acquisition.

Fig. 6. The best AROC of the IAS approach to UWB signal acquisition.

Fig. 7. The IAS AROC corresponding to hit set phases other than the LOS path when $G = 1$. 